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A follow-up study on the implementation of the recommendations of the committee on the undergraduate program in mathematics and other mathematics study groups within selected Massachusetts elementary school classrooms.

Mildred L. Vinskey
University of Massachusetts Amherst

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A FOLLOW-UP STUDY ON THE IMPLEMENTATION OF THE
RECOMMENDATIONS OF THE COMMITTEE ON THE
UNDERGRADUATE PROGRAM IN MATHEMATICS
AND OTHER MATHEMATICS STUDY GROUPS
WITHIN SELECTED MASSACHUSETTS
ELEMENTARY SCHOOL CLASSROOMS

A Dissertation Presented

By

Mildred Louise Vinskey

Submitted to the Graduate School of the
University of Massachusetts in
partial fulfillment of the requirements for the degree of

DOCTOR OF EDUCATION

September - 1970

Major Subject - Education

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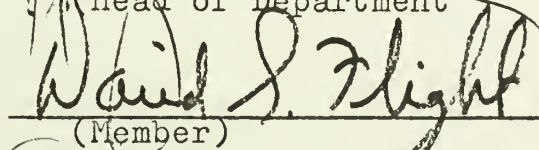
By


Mildred Louise Vinskey

Approved as to style and content by:


(Chairman of Committee)


(Head of Department)


(Member)


(Member)


(Member)

September, 1970
(Month) (Year)

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CHAPTER I

INTRODUCTION

Statement of the Problem

This study focuses upon the extent to which the recommendations of the Committee on the Undergraduate Program in Mathematics (CUPM), Panel on Teacher Training, of the Mathematical Association of America and various other mathematics study groups have been implemented in selected elementary schools of Massachusetts. Specifically, demographic data have been compiled relating to the three areas in which recommendations have been made by the above committees, namely, teacher preparation, instructional materials in use, and methods of instruction employed when teaching children mathematics.

Approximately nine years have elapsed since the CUPM published its report (1961), recommending that the pre-service training of elementary school teachers include the following mathematics courses, known as the CUPM Level I: (1:194)

1. A one-year course in the study of the structure of the real number system (6 semester hours).
2. A one-half year course in introductory algebra (3 semester hours).
3. A one-half year course in informal geometry (3 semester hours).

Various study groups and committees have been at work over a longer period of time on the improvement of the entire mathematics curriculum. If the recommendations of these groups are ever going to be followed, it seems reasonable to expect that significant progress in implementation should have been made by now. Therefore, the time seems right for making an assessment of this progress.

Since so-called "Modern Mathematics" has now become an integral part of our scientific and technological culture, the acceptance of new mathematics programs in our schools is an absolute necessity. Today, there are uses for mathematics that were unthought of a few years ago. Chemists and physicists have discovered new uses and interpretations for mathematics; biologists are applying mathematics to the study of genetics; businessmen are using mathematics in scheduling production and distribution; and sociologists are using complicated statistical ideas. Daily, we read about startling new scientific and technological research and developments, all of which are creating the need for an ever-increasing number of trained technicians and scientists. (2:15)

A wider and keener interest in mathematics must be developed among our youth not only to prepare them to be thinking and enlightened citizens in a complex world, but

also to lead more of them into the ever-growing number of scientific and technical positions required by our society. This is the enormous task which faces every teacher of mathematics today from kindergarten through college.

Various study groups and committees have decreed the type of curriculum that must be used to accomplish these goals. They have also decreed by the very form in which the materials have been presented that new methods of teaching must be used. Curriculum innovators have recommended changes in content but have not looked deeply into the complex problems which are involved in teaching mathematical ideas to children. Despite the fact that available empirical evidence does not indicate conclusively that heuristic methods of teaching and learning are more effective than traditional expository methods, these are the principles upon which most contemporary mathematics materials are based.

Textbook publishers have also changed the presentation of their materials to coincide with the thinking of the times. The researcher has examined many contemporary textbooks and has found that they are all based to some degree on heuristic methods of teaching and learning, often referred to as "discovery" techniques. A perusal of the prefaces of a few contemporary textbooks will corroborate

this statement. "To be truly modern, an arithmetic program must be modern in spirit. It must be a program that emphasizes inquiry, exploration, and discovery."; (3:vii) "The discovery approach is used. A great deal of responsibility for learning is placed on the learner, as he is constantly guided by sequences of carefully paced questions leading to discoveries of new concepts and relations."; (4:vii) "At every stage, children are encouraged to seek out and discover ideas for themselves, to look for interesting patterns and relationships, and to develop their own generalizations."; (5:3) "The authors of this series of textbooks have tried, through an inquiry-discovery approach, to foster a problem-solving attitude toward the study of all topics in elementary school mathematics." (6:T-12)

The acceptance of new mathematics programs in our schools raises the problem of providing for the training of the teachers who will be responsible for their implementation. An important reason for introducing new programs is to improve the teaching of mathematics. School administrators must abandon the idea that mathematics is an easy subject to teach. Mathematics can not be taught today with the same techniques that were used in the past. A great deal of time and effort are required to keep abreast of the changes in content, as well. (2:50)

Many school systems have introduced new programs, but what have they done to insure that their teachers are adequately prepared to use the new techniques? Have new teachers had courses covering heuristic methods of teaching contemporary mathematics? Have teachers who have been in service longer returned to college for refresher courses or even studied individually for professional improvement? Have summer institutes, pre-service and in-service workshops been provided? If so, have such workshops been one-time activities or are they being incorporated as an integral part of a continual curriculum improvement endeavor?

Several worthwhile programs have been described in the literature and are in current use in many school systems. Some are locally planned and funded; others are projects funded by private and government grants. The Madison Project, under the leadership of Robert B. Davis of Syracuse University, is an example of a complete program which, as its title indicates, "Pertains To The Interrelationship Of Mathematical Content, Teaching Methods And Classroom Atmosphere." This program provides for teacher education and curriculum revision within the school situation. Tape recordings and sound motion pictures of actual classroom experiences are used as a

means of teacher training, as well as a means of reporting to the teaching and mathematics professions. PhD. mathematicians work in classrooms with actual children and plan experiences for the children to help them learn the desired concepts. "The Madison Project does not use the common procedure of teaching mathematical ideas to teachers, and then leaving the teachers with the nearly impossible task of translating these ideas into suitable learning experiences for children." (7:107)

The University of Illinois Arithmetic Project, now associated with Educational Services, Incorporated at Newton, Massachusetts, under the direction of David A. Page, has a similar package of written materials and films designed for use in in-service institutes for elementary school teachers. School systems taking advantage of these materials meet weekly for nineteen weeks. Since many of the topics originally developed by the University of Illinois Arithmetic Project are included in the newer textbooks, these workshops can be of great help to teachers. The Project has given in-service institutes for teachers in Chelmsford, Concord, Framingham, Watertown, Newton, and Wellesley, Massachusetts. Regular teachers from kindergarten through seventh grade in Watertown and other schools in the Boston area serve as staff members of the Project.

Programs such as the two mentioned above are not readily available to all school systems. They serve the immediate areas in which they are located. In order for others to benefit, the deal is usually more expensive than many schools can afford.

Gerald R. Rising, Professor of Mathematics Education at the State University of New York at Buffalo, makes the following comments about the education of mathematics teachers: "Education programs for mathematics teachers are not only below any reasonably acceptable standard, but they are getting steadily worse!...While mathematicians and classroom teachers are working together to provide strong mathematics texts for students at all levels, the teachers who will be expected to implement these texts are nurtured on programs that are at best oblique to the tasks they face in the classroom." (8:296) Professor Rising blames this state of affairs on conditions existing in the colleges. Many campus schools are disbanding with the result that students are being deprived of participating in a teaching environment both within the college and responsible to members of the college staff, as well as further isolating college instruction from the practical world of the classroom teacher. Courses in mathematics methods are being reduced or replaced, so that al-

though students may leave college with strong mathematics backgrounds, they have received little in the way of professional assistance to enable them to transfer their mathematical knowledge into viable classroom procedures. (8:297)

Teacher education today is low on the status scale in the college scheme, because, as opposed to research, it is not in the self-interest of the college professor. Supervision of teaching has been curtailed, or is of such low quality, that it is useless. Many teachers in their first year of teaching are selected as cooperating teachers. Small budgets make it difficult to obtain and retain qualified methods teachers. (8:297)

Professor Rising further contends that nothing is being done to counteract this deteriorating development. He does not feel that CUPM recommendations for course work for elementary school teachers provide the answer to teacher training problems, because they are substituted for methods courses by professors who find it easier to lecture on mathematics than to help prospective teachers develop effective mathematical pedagogy. He accuses the National Council of Teachers of Mathematics of doing little or nothing to solve the dilemma and suggests that the President of this organization be urged to form a

committee to explore the national extent of this serious situation. (8:299)

These are the conditions which are influencing the teaching of mathematics in many of our schools today. Lofty goals and ambitions are being cited, but are they being attained? How can they be, if, as the literature seems to indicate, most elementary teachers do not have so much as a minor in mathematics, most experienced staff members have had no course work in the contemporary mathematics, and colleges and universities are still graduating teachers who are not prepared to teach the new programs?

Objectives

1. To determine the mathematics backgrounds and training of elementary school teachers employed in selected elementary schools of Massachusetts.
2. To determine the types of mathematics textbooks, materials, and manipulative equipment that are being included within selected elementary classrooms.
3. To determine methods of mathematics instruction that are being utilized by selected elementary school teachers.
4. To relate information acquired from 1, 2, and 3 above to recommendations made by CUPM and other mathematics study groups, as one means of assessing the progress

being made on these recommendations by local education agencies.

Hypotheses

This study is designed to test the following hypotheses, which are based upon the recommendations set forth by the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America and various other mathematics study groups:

1. That less than ten per cent of the elementary school teachers in Massachusetts have studied the twelve semester hours of mathematics required to meet CUPM recommendations.
2. That at least three-fourths of these teachers have never had a mathematics course of any kind since graduating from college.
3. That less than ten per cent of these teachers are allowed released time to pursue personal study, prepare lessons, consult with school specialists, etc.
4. That at least three-fourths of these teachers rely almost entirely upon one textbook when offering children mathematics instruction.
5. That of the elementary school teachers using contemporary mathematics materials and textbooks

in Massachusetts, less than ten per cent of them have had formal training in the use of these materials.

6. That at least three-fourths of these teachers adhere to expository (as contrasted with heuristic) methods of teaching children mathematics.

Background of the Problem

Significant changes in the elementary mathematics curriculum of the American schools have been brought about by influences which had their origins in the secondary schools during the past five decades. The most important of these was the statement of the Seven Cardinal Principles of Education by the Commission on the Reorganization of the Secondary School, appointed in 1914 by the National Education Association. The subsequent design of the secondary school curriculum was profoundly affected by this report. Previously, mathematics offerings had been few and college-oriented, but following the statement of the Seven Cardinal Principles, several new courses were added to the curriculum, including general mathematics, basic mathematics, consumer mathematics, shop mathematics, and commercial mathematics.

From 1920 to 1950, the utilitarian philosophy of the society of the times was reflected in the mathematics offerings of the schools. Skills and routine computations were stressed. Less emphasis was placed on theory and more on procedures that would be useful to the consumers, to government, industry, and commerce. Between 1930 and 1950, the demand for trained teachers of mathematics far exceeded the supply. Many mathematics majors had joined the war effort and left the field for better-paying jobs in industry. (9:4)

By 1950, many teachers were trying to teach at a level beyond that at which they had studied as students. Curriculum experts were becoming alarmed at the implications for the future of this undue emphasis on skills and preoccupation with the immediate usefulness of the subject matter. (10:47-48)

Between 1950 and 1960, the movement to improve the quality of the teaching of mathematics in American schools gathered momentum. Articles concerning classroom experiments and debates on the psychological and philosophical implications of curriculum revision flooded the professional journals of the 1950's. (9:8) In 1951, a Commission on School Mathematics, funded by the Carnegie Foundation and the United States Office of Education,

was established at the University of Illinois. The report of the University of Illinois Committee on School Mathematics (UICSM), adopted and published in 1959, was accompanied by a detailed description of the views of the Commission as to what would constitute a satisfactory mathematics curriculum. The work of this Commission resulted in the development of a sequence of entirely new mathematics courses for grades 9 through 12. (9:5)

Such was the state of affairs when the news of the launching of the first satellite (Sputnik) by the Soviet Union on October 4, 1957 was received. Factors of national security were injected into the picture to give additional impetus to the movement for more and better mathematics. School programs were widely criticized and pressure was applied to force school administrators to take immediate steps to remedy the situation. The federal government became involved through the granting of funds for curriculum development activities. College and university mathematicians became actively engaged in experimental projects and programs.

This climate prevailed when the School Mathematics Study Group, national in scope and representing the largest united effort for improvement in the history of mathematics, was organized.

The School Mathematics Study Group. The first meeting of the Group, which represented the combined thinking of psychologists, testmakers, mathematicians from colleges and industry, biologists, and high school teachers, was held in Chicago on February 21, 1958 and was sponsored by the National Science Foundation in order to survey the probable supply and demand of research mathematicians. (2:17) Views were expressed that one of the causes of the shortage of adequately trained persons was inadequate early schooling, and, although efforts were being made to improve school mathematics, these efforts were local in scope and under the auspices of single individuals and single institutions. A resolution was adopted at the Conference suggesting that the American Mathematical Society appoint a committee to seek funds and proceed toward the solution of the problem. On February 28, 1958, another meeting was called by the National Science Foundation at the Massachusetts Institute of Technology at Cambridge, Massachusetts, later referred to as the "Cambridge Conference". Mina Rees of Hunter College presided over the meeting, which was called to consider the existing mathematics curriculum in the schools of the United States. A decision was made that the Committee appointed by AMS would hold a writing session the following summer to prepare a detailed syllabus

for a model secondary mathematics curriculum beginning with grade 7, as well as write and publish monographs on mathematics of value and interest to secondary school students. Although it may seem that the logical procedure would have been to concentrate first on a strong elementary program, then build the secondary program on this foundation, there were reasons why this procedure was not practical. The improvement of instruction for the college-capable student was the primary objective of the movement to improve school mathematics. Students in high school needed to be able to take advantage of these new programs as quickly as possible. It was more practical to improve the secondary program, then work down through the grades with full realization that the entire sequence would require improved elementary programs, articulation between grade levels, and retraining of elementary teachers. (2:74)

A small sub-committee was selected to act for the Conference until the suggested Committee could be appointed by the President of the Society. It was very important that this Committee be selected by the President of the Society, since membership included every mathematician of stature in the United States. Active participation by these people would be easier to obtain, if the

Society created the Committee. Accordingly, Professor Richard Brauer, President of AMS, appointed a Committee of Eight, thus officially expressing interest in the mathematics curriculum of the schools and making it possible for research mathematicians to cooperate with high school teachers in the effort. Ultimately, approximately 100 mathematicians and 100 high school teachers cooperated in producing the materials. (2:17) Yale University assumed institutional leadership for the project. The National Science Foundation gave an initial grant of \$100,000. (9:13)

Although the School Mathematics Study Group is not necessarily representative of all curriculum study groups, it seems to have been the pioneer group which exerted the greatest influence on the contemporary textbooks and materials in use in our schools today. During the 1960-61 school year, 130,000 volumes of the revised edition had been sold. Orders for 226,000 books had been taken by July 1, 1961. Orders for 100,000 were on hand before the opening of the fall semester. Total sales for the 1961-62 academic year reached almost 500,000 volumes. These figures indicated that about 5% of the 10,000,000 student enrollment in the junior and senior high schools of the country were using SMSG materials. This percentage may seem small at first glance,

but it must be remembered that these books were paperbacks with no color and no illustrations to make them appealing. The sale of even this number of books gave encouragement that private publishers would find a ready market for satisfactory commercial replacements.

(9:124) In 1968, Professor E. G. Begle, Director of the School Mathematics Study Group, reported that 4,000,000 SMSG texts had been purchased. He stated that he had no way of knowing how many students and teachers have used these texts or other texts inspired by SMSG. (11:244)

In the field of mathematics, the textbook determines almost exclusively both what is to be taught and the sequence of the teaching of the material. (9:20) For this reason, a thorough examination should be made of the work of the SMSG to analyze the techniques of the group that has had such a far-reaching influence on the materials and teaching procedures now included in most contemporary textbooks.

The School Mathematics Study Group is a unique organization with somewhat different aims and objectives than other groups making studies in the teaching and learning of mathematics. The main purpose of the SMSG texts is to serve as a model and as a course of suggestions and ideas for the authors of this variety of textbooks. Textbook

writers may adopt, expand, and improve them for their own purposes, so long as credit is given and no endorsement is implied. (12)

Each text prepared by the Group passes through three editions: the preliminary edition, the revised edition, and the sample text edition (no further revision by SMSG seems to be required). All SMSG materials are published at cost under support of the National Science Foundation, so no free sample copies can be sent to individuals or organizations. A non-profit press is contracted to publish the sample texts. The word "sample" is used in the sense that these are samples of mathematics which SMSG feels can and should be taught. The plan was to allow the sample text editions to go out of print when a sufficient amount of commercially available textbooks incorporating SMSG materials appeared on the market. (12)

In 1961, SMSG decided to continue its projects indefinitely in close collaboration with classroom teachers and research mathematicians. The Advisory Board adopted the following bylaws: "The primary purpose is to foster research and development in the teaching of school mathematics - a continued review of the mathematics curricula in the schools as an aid in the design and selection of promising experiments. It will also consist in part of an analysis of the results of experimental teaching as an

aid in judging whether the objectives of the various programs are being achieved, but the work should consist primarily in the development of courses, teaching materials, and teaching methods. Special provisions need to be made so that students at various ability levels can be taught in appropriate style and at appropriate paces. It should be a bold experiment with courses differing sharply from the present practice in their style, or their content, or both." (13)

The improvement of elementary school mathematics was the sixth project undertaken by the School Mathematics Study Group. This project stressed providing materials with increased emphasis on concepts and mathematical principles with grade placement of topics, the introduction of new topics, particularly from geometry, and supplementary topics for the better student.

Experimental centers were established to test the texts and materials produced by SMSG. These centers were established in cities or other localities under the supervision of local people who acted as chairmen. Their responsibility was to secure suitable teachers and classes for a try-out process. Each center was assigned a consultant, usually a college mathematician who met periodically with the teachers of the experimental classes. The "center concept" for testing was highly successful. This type of

organization decentralized the details of distribution; local people cognizant of the customs of the area were on hand to cope with minor problems; and, since the centers were located in the vicinity of some college or university, the consultants were readily available. (2:46)

The SMSG curriculum is innovative in the sense that it introduces subjects not previously taught at certain levels. For example, geometry is introduced in a simple form in the primary grades. In this respect, it follows Jerome Bruner's philosophy that the foundations of any subject can be taught to any child at any age, provided they are presented in language the child can understand and at a level parallel to his knowledge and experience. It departs from the traditional curriculum by introducing such items as probability ratios, bases other than the decimal, use of exponents, statistical ideas, and geometric functions in the elementary grades.

The Brunerian influence is seen again in the structure of the materials used in the texts. The underlying principles of the subject are used as a basis for recognizing subsequent problems as special cases of the original idea and applying the learned knowledge to the new problem. His ideas of sequence are also followed, but, at times, are apt to be somewhat "lock-stepped" in that each idea must be closely followed, in order not to lose

the continuity of the subject matter. In other words, the text must be closely adhered to as far as specific aspects of the subject units are concerned.

The researcher was particularly surprised to discover that the objectives in all of the literature perused were loosely phrased and not at all definitive: "The world of today demands more mathematical knowledge on the part of more people than the world of yesterday, and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; an understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship." (9:49) The term "intelligent citizenship" is not defined. The Group reasons that, since no one can predict his future profession with certainty, much less tell which mathematical skills will be required by a given profession, it is important that mathematics be so taught that students will be able to learn in later life the new mathematical skills which the future will surely demand of them.

Logical reasoning and critical thinking are supposed to be the outcomes of using these texts. However, the teacher is not told how to assure the attainment of these worthwhile objectives, but is left to his own devices to implement them. The teacher's text does not give any aid

to the teacher in deciding what behaviors to seek or how these behaviors could be learned. To quote Taba: "A platform of general objectives, no matter how well defined is still an inadequate guide for the specific aspects of the curriculum...these general objectives need to be translated into more specific ones...those in charge of curriculum development must pay some attention to the process of implementing the general objectives in all its steps..." (14:228-29)

To sum up, the strongest criticism that can be made of these texts is that they are just texts, not an organized curriculum according to modern standards of curriculum building. The objectives are vague, with no definitions of the outcomes and attitudes that are expected; all responsibility for implementing the objectives, such as they are, is declined by the Group and placed squarely on the shoulders of the school system deciding to use the materials; and, lastly, no valid measuring instruments have been devised to evaluate the effectiveness of these texts or their superiority over traditional mathematics texts.

Contributions of other groups. While the School Mathematics Study Group was making extensive textbook revisions, other programs for improving mathematics in the elementary school were being widely publicized. Although each of the new programs had unique features, the following characteristics were common to all:

1. Mathematicians assisted in the development of materials.
2. Great emphasis was placed on the use of discovery techniques.
3. The importance of correct terminology to identify mathematical ideas and the use of precise terms were stressed.
4. Grade placement of topics was readjusted.
5. Topics not typically taught in elementary school were included.
6. Increased emphasis was given to the structure of mathematics, its laws and principles, patterns and sequences.

Although advocates of these plans are not in complete agreement on all of these elements, there is general agreement among them on four major assumptions: that children should be taught a standard vocabulary for mathematics in the primary grades and correct names for concepts should be taught at every level;* that grade placement of skills should be reorganized; that skill in computation should be accompanied by an understanding of the process used, its purposes, and the laws which govern it; and that the child should be allowed to assume an active role in the act of learning by discovering and developing mathematical ideas by himself. (15:34)

* Current thought is not entirely in agreement with the first assumption.

The Cuisenaire Numbers-In-Color Plan. This system devised by Georges Cuisenaire, a Belgian schoolmaster, involves the manipulation of colored rods with number length equivalents. In 1953, Dr. Caleb Gattegno, mathematics lecturer at the London Institute of Education, recognized the value of Cuisenaire's techniques as a means of implementing mathematics and the plan was adopted in many of the British schools. The rods are now widely used in the United States, especially in the primary grades, although programs for use with the colored rods are available through the ninth grade. The method is now being used to teach the deaf, the blind, and the mentally retarded. Cuisenaire's plan is a non-structured learning activity based on pupil discovery, discussion, and evaluation of children's ideas. (15:35)

The Madison Project (1957). This project, under the direction of Dr. Robert B. Davis of Syracuse University, was an experiment designed to use new methods and materials for stimulating greater interest in the study of mathematics. The program is a supplemental one, with a minimum of one class period per week assigned to the material, although teachers may use more time whenever feasible. The program is not an accelerated one but is recommended for any class of normal pupils heterogeneously grouped by age and background. The lessons, called "Creative Learning

Experiences", are directly related to one or more of the fundamental mathematical concepts included in a special list. The child must have an active role and, as much as possible, an autonomous decision-making role. Lessons are worked out by PhD. mathematicians in the classroom that are appropriate to the particular children being taught as to age, needs, backgrounds, and previous experience with mathematics. (7:3-5)

The University of Illinois Arithmetic Project (1958).

This project, under the direction of Dr. David A. Page, also had as one of its important aims the development of a mathematics program that would prove interesting and stimulating to children, as well as to improve the methodology of instruction in elementary mathematics through in-service institutes, teacher reeducation seminars, and project publications.

The Illinois Project uses frames, but instead of describing the shapes (the number in the triangle, etc.) as the Madison Project does, arbitrary names with phonetic spellings are given to each frame (a square is ekks, a triangle is wye, and an upside down triangle is zee). Through use of the frames, children are led to discover properties of numbers and rules of operation and to make generalizations about number relationships. (15:50)

The Greater Cleveland Mathematics Program (1959). A

large number of public, private, and parochial schools in the Greater Cleveland (Ohio) area organized to promote educational research and the improvement of instruction in their schools. Dr. Bernard M. Gundlach directed the project.

This program differed from others in that its directors believed the elementary mathematics program must be foundational. Innovations in mathematics should provide for a broader curriculum in secondary mathematics, and to accomplish this, adjustments must start in the lower grades.

GCMP made an important contribution by making the public aware through many media of mass communication of the need for changes in elementary school mathematics.
(15:56)

The Stanford Project (1959). "Sets and Numbers", a program developed by Dr. Patrick Suppes of Stanford University for use in the primary grades, was based on his belief that sets are more concrete than numbers and that the introduction of sets permits mathematically exact definitions of the relations between concrete objects and Arabic numerals. (15:41) Algebra is introduced in the first half of the first year and equations are balanced with letters replacing numbers. Aspects of geometry, including line segments, points, plane figures, perimeters and areas are some of the topics studied in the first and

second grades. (15:46)

The Nuffield Project (England) is also worth noting. See page 117.

Significance of the Problem

There are definite implications in contemporary mathematics that have a significant bearing on the manner in which arithmetic and geometry should be taught to elementary school children. Teachers must be thoroughly prepared in the areas of mathematics in which they are giving instruction. The effectiveness of any program depends on the teacher and the method of teaching rather than on any textbook however good it may be.

Research has shown repeatedly that elementary school teachers lack mathematical background. Leaders in mathematics education have long recognized this lack of background as a major obstacle to the improvement of mathematics instruction. (16:137)

Although CUPM recommendations produced new college courses and new textbooks, elementary school teachers frequently complain about the inadequacy of current college courses. (16:137) The evidence of this dissatisfaction should alert the profession to the pressing need for careful consideration of the problem. One of the main causes for dissatisfaction stems from the irrelevancy be-

tween what is studied in the college courses and what is taught in the classroom. When discussions of aspects of teaching mathematics and working with children are divorced from teaching the mathematics content, many students whose academic preferences lie elsewhere are deprived of their main source of motivation. "Improved content" courses can not be taught in isolation. (17:59)

Twenty-five leaders in mathematics education in all parts of the country were asked to express their opinions on issues and directions of elementary school mathematics. Included in this group were college and university mathematicians, experts in elementary education, state departments of education personnel, and supervisors and teachers at elementary and secondary levels. The needs most frequently mentioned in the twenty-two replies received were:

1. Improved programs of in-service and pre-service education in mathematics for elementary school teachers. The blame for deficiencies in this area were placed directly on the colleges and universities and school administrators, not on the classroom teacher.

2. Increased use of teachers with some specialization in mathematics, with helping teachers, team teachers, and at least one specialist in each school. (18:23-24)

Definition of Terms

Contemporary mathematics. This term is used to denote the kind of mathematics currently being taught in the schools of the United States in the second half of the twentieth century.

The terms "modern" mathematics and "new" mathematics, ambiguous and misleading phrases prominent in the literature today, infer that this mathematics has never been taught before. One of the most fundamental errors being made concerning the revolution in mathematics is that it is one in material when, in reality, it is one in method.

Heuristic method. "Discovery" is the term most commonly used to refer to this method. The heuristic method is the process of leading the pupil by skillful questioning to find the desired knowledge by himself. The individual student applies the scientific method of inquiry in the classroom.

This method is dramatically differentiated from the "tell and do" method by feedback from the student's behavior to the teacher. Both teacher and pupil make hypotheses from available data, rejecting or accepting them in terms of new data which become available.

Conceptual mathematics. That knowledge of mathematics which allows one to give reasons for various ways of com-

puting is known as conceptual mathematics. It implies knowledge and understanding of the basic concepts, principles, laws, patterns, sequences, ideas, and structure of mathematics.

Pre-service training. This training includes all college-level activities pertaining to mathematics received prior to becoming a classroom teacher.

In-service training. This training includes all college courses, workshops, summer institutes, or other training received while being regularly employed as a classroom teacher.

CHAPTER II

RELATED RESEARCH

Introduction

This chapter is concerned with the summary and evaluation of research studies in the following areas:

1. The relative merits of heuristic methods of teaching and learning as contrasted with expository methods.
2. Studies conducted by SMSG Experimental Centers and others comparing the performance of pupils using SMSG materials based on discovery methods with that of pupils using traditional materials.
3. The adequacy of the mathematics preparation of prospective elementary school teachers.
4. The attitude of elementary school teachers toward the teaching of mathematics.
5. The effectiveness of pre-service and in-service programs as a means of strengthening understandings of elementary school teachers in the contemporary mathematics curriculum.
6. Cooperative school and college relationships that are contributing to the improvement of mathematics instruction.

7. The extent of implementation of CUPM Level I recommendations.

Heuristic Methods of Teaching

Prior to making a study of the related research on teacher preparation, an examination will be made of the various aspects of the heuristic method in order to provide clearer insight into the origin, characteristics, and importance of this concept.

Heuristic methods of teaching and learning are not new techniques. Their origin can be traced as far back as Plato when he had Socrates say: "Do you observe, Meno, that I am not teaching the boy anything, but only asking him questions?" (19:121)

In 1847, David Page, the first principal of New York State's first normal school, stated in his Theory and Practice of Teaching that "there is a great satisfaction in discovering a different thing for oneself...the teacher should be simply suggestive." (19:121)

In 1897, Charles and Frank MacMurray wrote in their book, The Method of the Recitation, that "the child is expected to conceive these answers himself; he is systematically required to make discoveries...to judge what might reasonably follow from a given situation, to put two and two together and declare the result." (19:121)

In 1905, in The Educative Process, William Bagley maintained that "Whatever the pupil gains, whatever thought connections he works out, must be gained with the consciousness that he, the pupil, is the active agent - that he is in a sense at least, the discoverer." Although this quotation was attributed to Bruner at the Woods Hole Conference of the National Academy of Sciences in 1959, it originated with Bagley. (19:120)

The word "heuristic" is derived from the Greek word "heuriskein" which means to "discover". This accounts for the method being commonly referred to as the "discovery" method. Heuristic methods have been emphasized in books on methods of teaching mathematics since 1906. (19:122)

A discovery approach is being used when material is presented to pupils in a manner that challenges them to look for patterns and relationships and to draw logical conclusions for themselves. Discovery teaching precludes wordy explanations and memorization of rules. Instead, emphasis is placed upon the forming and testing of hypotheses.

There is no one discovery method. Bittinger visualizes the discovery method as a combination of many methods. He recognizes four distinct classifications:

1. The Inductive Method - the child is given various

examples to lead him into the knowledge of a generalization.

2. The Deductive Method - the pupil attempts to find a proof of his own from an accepted general statement.

3. The Variation Method - the pupil changes elements of the data or conclusions, or both, in order to obtain new data or new conclusions.

4. The Non-Verbal Awareness Method - the pupil is not required to verbalize the generalizations being taught. (20:141)

Massialas believes that teaching through discovery requires a classroom climate that will encourage wide student participation. To accomplish this goal, the teacher must assume a wide variety of roles: As a planner, he collects and prepares materials and organizes the spacing and sequence of these materials; as an introducer, he introduces new learning experiences with appropriate materials to stimulate inquiry and discussion; as a questioner and sustainer of inquiry, he encourages the students to find alternatives for problem solving and to defend their positions; as a manager, he oversees the entire operation and leads students to plan and execute inquiries of their own; as a rewarder, he praises their success when hunches pay off in the free exchange and

testing of ideas, thus furnishing high levels of motivation and greater student participation; and, finally, as a value investigator, he emphasizes that students must be able to defend value judgments publicly. (21:41)

Ausubel recognizes that the discovery method has a defensible rationale and that it does have a value as one of the many techniques available to the teacher. However, there are times when its use is neither feasible nor appropriate. In his words: "The proposition that every man must discover by himself every bit of knowledge he wishes to possess is a repudiation of the very concept of culture. The most unique attribute of human culture is precisely the fact that accumulative discoveries can be transmitted to each succeeding generation and need not be discovered anew." (22:291) According to Ausubel, the success of such programs as the University of Illinois curriculum study in mathematics can be attributed to two reasons: first, students need to be reeducated, because they do not have a sound meaningful grasp of the basic facts of mathematics in the first place; and, secondly, as the program develops, the element of discovery is gradually lessened, until it is eventually given only token recognition. (22:290-302)

On the other hand, Max Beberman contends that the

use of the discovery method in the UICSM curriculum is justifiable, because the pupil is attracted to the "what would happen if" question, regardless of its practicality from an adult standpoint. Kersh takes issue with Beberman on this and points out that there is little in the literature relating to this reaction to discovery learning. He regards the instruction provided by the teacher as an important contributing factor to learning - more so than what is withheld. (23:417)

Bruner argues that through discovery learning the pupil develops the ability to organize information for later application. He becomes less dependent upon the external motivation of parents and teachers. He is self-motivated to attempt to solve problems and is intrinsically rewarded if he succeeds. (24:22)

Most educators agree that the discovery approach, with all of its variations, is the preferred method for teaching mathematics today.

Research on Heuristic Methods

Mathematics educators often use findings and conclusions of studies such as the following as a basis for recommending de-emphasis on drill and emphasis on teaching methods which encourage the forming and testing of hypotheses.

Winch (1913) found in what appears to be the first experiment on discovery learning that better retention is obtained from expository learning and better transfer from inductive learning. (25:59)

Hendrix (1947) compared "tell and do" methods with inductive methods and found that the highest transfer effects were achieved by students who were taught by the un verbalized awareness method. Lowest transfer effects were achieved by students who were taught by expository methods. These findings seem to indicate that the key to learning is sub-verbalized; the organism must be affected in some way before it has any new knowledge to verbalize. (26)

Haslerud and Myers (1958) confirmed the findings of Hendrix. They concluded that "principles derived by the learner solely from concrete instances will be more readily used in a new situation than those given to him in the form of a statement of principles and an instance." (27)

Luchins and Luchins (1950) concluded that "tell and do" methods tend to develop fixations, not adaptive responses. A pupil may know rules and formulas, yet not be able to apply them or to determine what method should be used in a particular instance. (28)

Miller (1951) concluded that "It seems likely that in being trained to utilize sure methods of work, pupils tend to approach all new situations in this way, thus failing to show the same flexibility in attack as do other pupils whose formal training emphasizes finding alternative methods of solution." (29)

The validity of the results of most of the earlier experiments are questioned by Hermann, because these studies were designed to investigate aspects of learning which were peripheral to the discovery method. Due to the complex nature of discovery learning and the lack of significant findings, the following conclusions are regarded by Hermann as being tentative only:

- (a) Better retention is obtained from ruleg (rule-example) learning.
- (b) Better transfer is obtained from discovery learning.
- (c) Discovery learning is relatively more effective as the difficulty of the transfer task increases.
- (d) Discovery learning is relatively more effective as the period of time between learning and testing on a transfer task increases.
- (e) Discovery learning is relatively more effective when the learning task involves material such as that taught in schools.
- (f) There may be a tendency for discovery learning to be relatively more effective when the background knowledge in a subject is limited.

- (g) The discovery method is relatively more effective for low ability groups than for high ability groups.
- (h) After material has been learned by a discovery method, immediate verbalization or further learning adversely affects the original learning.
- (i) In the discovery method, a reasonable degree of guidance is better than little guidance.
(25:65-66)

SMSG Research Studies

During the 1961-62 school year, each Experimental Center conducted a testing program involving children using SMSG texts. In one study, a total population of 600 pupils in grade 4 and 1200 pupils in grade 5 was tested. The children were found to be above average in terms of the index of arithmetic aptitudes and the estimated IQ, based on the scores of two administrations of SRA Arithmetic Achievement Tests. The results indicated that students in SMSG classes do just as well on standard tests of mathematics skills as students in conventional courses. At the same time, the students are exposed to and learn a number of concepts not available in conventional courses.

A special instrument called "Ideas and Preferences Inventory" was devised and used to measure attitudinal factors. The scores indicated a tendency, though not a

marked one, in the direction of favorable attitudes toward mathematics. Mean scores for 5th grade pupils who had already used SMSG sample texts in grade 4 were indicative of no more favorable attitudes toward mathematics than the mean scores of 4th grade pupils using SMSG texts for the first time. Both boys and girls, both 4th and 5th grades, showed a slight tendency to have more favorable attitudes toward mathematics at the end of the year than at the beginning.

It is very difficult to tell whether this change in attitude was as little as the instrument seemed to indicate, or whether the instrument was insensitive to more marked attitudinal changes that really did take place. There was no way to judge the validity of the instrument. Informally reported reactions from many teachers would tend to support the latter hypothesis, but there is no evidence to give an answer with reasonable confidence.

(30)

In another study conducted by the Minnesota National Laboratory, ten pairs of 4th grade classes in the vicinity of Minneapolis and St. Paul were chosen to participate in an evaluation of SMSG. In each pair, one class was experimental, the other controlled. The two classes in each pair were taught by two different teachers. Students in

all pairs of classes matched on the basis of IQ and achievement tests, and, to some extent, by the teacher's difference between the experimental and control classes. From a record of educational background and teaching experience, there was some indication that the experimental teachers had a slightly better background in mathematics.

To compare the progress of a year's teaching, the STEP Test 4a was given to all classes in September, 1961 and May, 1962. They were also given the Differential Aptitude Test in February, 1962.

Those who participated in the experiment believed that it gave evidence for the superiority of material such as SMSG over traditional texts. There was no difference in the two groups in the progress in mastering traditional work. However, the experimental group spent considerable time on units not reflected in their performance on STEP 4a.

The experiment was based on too small a sample and spread over too short a time interval to be conclusive.
(30)

In 1960, a study was carried on by the Educational Testing Service in seventy-five schools in the United States with the fundamental purpose of comparing the achievement of students in the SMSG courses with that of

students in non-SMSG courses.

Two groups of teachers (approximately 30 in each group) were selected at random. One included teachers willing to teach the SMSG curriculum for the first time, using conventional mathematics instruction; the other included teachers willing to teach the SMSG curriculum for the first time, using mathematics instruction based on SMSG materials. The students of both groups of teachers were administered common tests of scholastic aptitude and knowledge of mathematics in the fall of 1960 and common tests of traditional mathematics and SMSG mathematics in the spring of 1961.

The results indicated that students exposed to conventional mathematics have neither a pronounced nor a consistent advantage over students exposed to SMSG materials with respect to the learning of traditional skills. The tests showed that students exposed to SMSG instruction acquired pronounced and consistent extensions of mathematical ability beyond that developed by students exposed to conventional mathematics instruction. (31)

None of the studies conducted at the Experimental Centers seem to indicate any great difference in computational skills whether traditional texts or SMSG texts were used. The difference may very well lie in the conceptual skills and behaviors for which no valid instrument of

measure has yet been devised.

The measurement aspect of the contemporary mathematics curriculum makes investigation of its effectiveness extremely difficult. Valid tests are needed for measuring objectives that are independent of content, such as problem solving, logical reasoning, creativity, and attitudes.

Pate (1965) made a study to determine whether differences in interaction patterns existed between the SMSG program and the traditional mathematics program. An important objective of the study was to find out whether teachers were using discovery methods in their mathematics teaching. Twenty classes studied each type of program. The following conclusions were drawn: Significantly more SMSG teachers used analysis and comprehension questions, as well as more divergent questions to elicit creative and spontaneous responses; teachers in traditional classes relied more on cognitive memory than any other operation; the small amount of opinion and synthesis questions used by SMSG teachers indicated that full implementation of the processes of inquiry and discovery had not been developed; and, although there was a significant difference between the two programs, only a small proportion of student and teacher responses related to the system of inquiry and discovery.

Pate's study contains an important implication for

the teaching of mathematics, namely, that "mathematical content should not be divorced from the methodology associated with the system of inquiry and discovery."
(32:21-24)

Mathematics Training of Prospective
Elementary School Teachers

Melson (1965) concluded from a study of college preparation for teachers of contemporary mathematics that most of the 41 teachers tested were "not adequately trained in college to teach the elementary mathematics concepts which have been recommended for grades 1-6 by The National Council of Teachers of Mathematics, the state departments of public instruction, and the authors of recently published mathematics textbooks and materials."
(33:53) On a 33 item test containing items that are basic to most contemporary mathematics programs, two-thirds of the teachers tested correctly answered less than 50% of the questions. (33)

Smith (1965) attempted to corroborate Melson's results by administering the 26 items published in Melson's study to two groups of elementary education majors who had just begun a methods course in the teaching of arithmetic taught in the education department. The results of the pretest were similar to those of the original study,

but there was a significant difference between the pre-test and the posttest. Although the 80 students tested by Smith had knowledge similar to that of the original group studied by Melson, it can be assumed that the significantly higher scores on the posttest were influenced by a review of certain concepts and discussions of the relationship of these concepts to the objectives and methods of elementary school mathematics. (34:202) This study illustrates very well the importance of effective methods courses.

Gibney, Ginther, and Pigge (1967-69) investigated the problem of whether prospective elementary school teachers without any teaching experience do better on a test designed to measure basic mathematical understandings than in-service teachers. The test included 65 items and was administered to two different groups: the first included students at Bowling Green State University (Ohio), the University of Toledo, and Eastern Michigan University who had completed at least one three-semester-hour course in mathematics covering the real number system and topics in geometry; the second included in-service teachers with about the same education and experience patterns as the pre-service group. One-thousand eighty-two tests, measuring understandings in seven areas, were administered. The scores favored the pre-service teachers at 1st, 2nd,

3rd, and 4th grade levels. There was no significant difference between the two groups at kindergarten, 5th, 6th, 7th, and 8th grade levels.

On the whole, pre-service teachers with no experience scored significantly higher than the in-service teachers, indicating the need for different treatments in the mathematics education courses designed for these two groups. (35)

Reys (1966) reported that the mathematics preparation provided at the University of Missouri at Columbia did not satisfy the minimum requirements proposed by the Committee on the Undergraduate Program in Mathematics. His findings revealed that approximately one-third of the recent graduates were dissatisfied with their pedagogical preparation; a large per cent of the recent graduates with grade averages of A, B, and C rated the program ineffective; more than three-fourths of the recent graduates desired additional training in mathematics. Reys raised the question of whether these results were unique to the University of Missouri or whether they were characteristic of the mathematics preparation in other institutions. (36)

Professional textbooks are a valuable source of help for the prospective teacher. Cruikshank conducted an investigation in an attempt to discover whether methods textbooks have kept abreast of the changes in the mathe-

matics curriculum, or whether they are substantially the same as they were for the traditional mathematics. An analysis of data obtained from six textbooks used in pre-service mathematics courses, randomly selected from a random survey of teacher training institutions, failed to answer the question: "What is modern mathematics insofar as the elementary schools of this nation are concerned?" (37:480) Answers to questions on 10 items in the survey differed little from those published between 1930 and 1960. There was no consistent agreement on the objectives set forth in the books. The books varied considerably on the amount of emphasis placed upon commonly discussed topics. Contemporary thought on the elementary school mathematics curriculum was very similar to that found in pre-service textbooks and professional yearbooks published since 1930. The analysis of these professional textbooks did not yield any systematic direction for contemporary mathematics. (37)

Hardgrove found (1964) that in a survey of 906 colleges (762 responses) 22.4% required no mathematics of elementary school teachers, 68.9% required the equivalent of 4 or less semester hours of mathematics, and 55.6% offered no mathematics courses specifically designed for elementary school teachers. (38) These figures indicate

dramatically that three years after the CUPM report was published, nothing had been done to implement the Level I recommendations in the colleges and universities involved in this study.

Prospective Teachers' Attitudes Toward Mathematics

Smith (1964) compared prospective teachers' attitudes toward mathematics with those reported by Dutton in 1954. One-hundred twenty-three students rated themselves on an 11 point scale ranging from "strongly against" to "strongly for" on 23 attitude statements. While the data were in agreement with Dutton's findings that strong attitudes toward mathematics are developed in all stages of our educational system, more than one-half of the students in the study chose the elementary school years as the time when their feelings toward mathematics developed. There was also agreement with Dutton that many students preferred some areas of mathematics over others. Either neutral or favorable attitudes toward mathematics were indicated by 88.6% of the students in this study as compared with 79.5% of Dutton's subjects. (39)

Kane felt that the revolution in mathematics had possibly caused Smith's study to reflect a substantial Hawthorne effect among prospective elementary school teachers.

Questions might have been answered on the basis of socially acceptable behavior rather than underlying attitudinal dispositions. He noted that the responses in 1964 did not show a trend toward more positive attitudes over earlier responses. Kane made a study for the purpose of devising a "neutral" instrument on attitudes of prospective elementary school teachers toward mathematics by including items which exhibited no preoccupation with mathematics. Respondents were asked to rank-order English, Science, Mathematics, and Social Studies in response to six statements. The questionnaire was administered to 58 elementary education majors at Purdue University (Indiana) at the close of the student teaching period. Attitudes toward mathematics were found to be relatively high. Mathematics had the highest attitudinal status among the teachers who planned to teach in grades 4 through 6. Teachers with unfavorable attitudes indicated a preference for teaching in the primary grades. (40)

A study by Reys and Delon at the University of Missouri at Columbia during the 1965-66 academic year focused upon the overall mathematics preparation program for education majors. Dutton's Attitude Scale containing 15 questions reflecting attitudes toward arithmetic was administered to 385 students prior to and following one of

the three courses included in the program. Approximately 60% of the students in the study expressed favorable attitudes toward arithmetic (55.58% in the pre-course inventory as compared with 58.70% in the post-course inventory, with the difference not being statistically significant). Unlike the subjects in Smith's and Dutton's studies, the greatest per cent of students indicated that their feelings toward mathematics were developed in the junior high school. The fact that the mathematics preparatory course produced only a small change in attitude might be explained by the short duration of the course. A continuous mathematics program for a longer period of time might result in a large scale improvement of these feelings that had become deep-seated over the years. Favorable attitudes toward mathematics must be fostered from elementary school through college. (41)

Pre-Service and In-Service Programs

Two sections of students in a methods class at the University of California (Los Angeles), most of whom would enroll in practice teaching during the spring or fall semester of 1965, were involved in a study by Dutton. The students were required to teach three hours each week in nearby elementary schools. Comprehensive tests were given in conceptual mathematics and students were urged to seek

additional help in areas in which the tests indicated they had difficulty. Responses on the pretest ranged from 37-96%, with a median score of 77%; on the posttest, 69 out of 80 students scored 86%, with a range of 66-100% and a median score of 91.84%.

This study points toward marked progress in the mastery of modern mathematics concepts when instruction is adjusted to individual needs. The college must not operate on the assumption that brief pre-service courses will enable elementary school teachers to understand and participate in meaningful teaching of the new programs. (42)

Dutton and Hammond (1966) studied two different instructional plans designed to help teachers understand contemporary mathematics:

1. A workshop conducted by a college professor of mathematics.
2. A district-planned in-service workshop, using school staff for instruction, with no textbook and a variety of instructional materials.

The researchers hypothesized that the workshop with the structured program would result in better teacher understanding of basic mathematics concepts and more favorable teacher attitudes.

Both groups showed significant improvement in their understandings of concepts, but, on the posttest, the

amount of gain in mean score points of the district using its own staff was almost double that of the structured workshop. Probably, the unstructured workshop gave individual teachers more opportunity to work on specific difficulties.

Again, this study emphasizes the importance of using diagnostic tests to pinpoint specific weaknesses teachers have in understanding concepts. Experts in mathematics should be used to help correct these weaknesses rather than to conduct structured classes. (43)

Many schools have adopted new textbooks before teachers were trained. Harper wanted to show that there was enough difference in basic mathematical understandings to warrant an in-service program (Colorado). A random sample of 100 elementary schools was chosen. The findings were as follows:

1. The group who had a course in modern mathematics scored significantly better on the test. However, after the testing of the first hypothesis, a control of 6 hours or more of college mathematics was used on succeeding hypotheses, because of the great differences in mathematics backgrounds of the teachers in both groups.

2. The group who had no modern mathematics but had 6 or more hours of college mathematics performed signifi-

cantly higher than those who had no modern mathematics and less than 6 hours of college mathematics.

3. Those who had no modern mathematics but had 6 or more hours of college mathematics performed significantly better than those who had modern mathematics but had less than 6 hours of college mathematics.

4. Those who had training in modern mathematics but had less than 6 hours of college mathematics performed at significantly higher levels than those who had no modern mathematics instruction but had less than 6 hours of college mathematics.

5. Those who had modern mathematics and also had 6 hours or more of college mathematics scored significantly higher than those who had modern mathematics but had less than 6 hours of college mathematics.

6. Those who had both modern mathematics and 6 hours or more of college mathematics did significantly better than those who had no modern mathematics and had less than 6 hours of college mathematics.

7. Those who had modern mathematics and 6 hours or more of college mathematics did significantly better than those who had 6 hours or more of college mathematics but had no training in modern mathematics.

Teachers profit by training in college mathematics,

as indicated by these findings. Those who had 6 hours or more of college mathematics scored significantly higher in every comparison. Even those who had 6 hours or more of college mathematics performed better on the test when they also had training in modern mathematics.

The scores ranged from no correct responses for two teachers to 55 correct out of a possible 61. (44)

This study implies that teachers need training in modern mathematics; that teachers benefit from college training in mathematics; that elementary school teachers should have in-service training which incorporates basic mathematics and modern mathematics; and that every teacher should study a methods course which pays considerable attention to the teaching of these concepts in the elementary school.

Cooperative School and College Relationships

If improvement is to be realized in the training of elementary school teachers to teach contemporary mathematics, true cooperation must exist between the schools and the colleges. All aspects of pre-service and in-service training must be fused together - neither can operate effectively independent of the other.

Houston (1961) reported on a program which involved several departments of the University of Texas in cooperation with the Austin public schools. Five weekly sessions (1½ hours each) were planned and presented by a teaching team from the University. Instruction in both content and method was given to 253 participants, including 43% primary teachers, 43% intermediate teachers, 8% junior high school teachers, and 6% special teachers or administrators. Seventy-five per cent of the group had more than four years of teaching experience; 90% had at least one course in mathematics or in the teaching of mathematics. This in-service program used the team teaching approach, television, lectures, question-discussion, and written materials.

The teachers highly favored the team approach. Eighty per cent of the teachers reported that they used the materials from the series in their classroom teaching. Those with four or more years of teaching experience rated the series significantly higher than those with less. They also rated the lectures higher. Those with less teaching experience rated television and question-discussion sessions significantly higher. The study gave evidence of a relationship between the effectiveness of various media to teaching experience or age, or perhaps to both, thus emphasizing once more the need for individuali-

zation of instruction. (45)

Catmull described a program devised for the Granite School District (Salt Lake County, Utah) during the 1965-66 academic year. The program was initiated, because a new series of mathematics textbooks had been introduced. A large majority of the elementary school teachers had no mathematics courses in college and felt very insecure trying to teach the unfamiliar material.

Two paid mathematics teachers from the district worked each morning with two University of Utah instructors to plan and prepare materials for the class of 80 teachers which met each afternoon for three hours over a period of five weeks. Two of the three hours were devoted to lectures and discussions and one hour to supervised study.

When teacher reactions were checked at the end of the course, two-thirds of the participants expressed a desire to take another course in 1968. Statements such as the following on the evaluation sheet indicate that proper training can change the attitude of teachers toward mathematics: "This is the first year I have enjoyed teaching mathematics. Now it is my favorite subject." (46)

Extent of Implementation of CUPM Recommendations

The Level I recommendations of the Committee on the

Undergraduate Program in Mathematics of the Mathematical Association of America, as previously stated, proposed twelve semester hours of mathematics as the minimum requirement for elementary school teachers. The recommended courses included six semester hours in the study of the real number system, three semester hours in introductory algebra, and three semester hours in informal geometry. During the five years following the publishing of the recommendations, the CUPM conducted a series of conferences with mathematicians, educators, administrators, classroom teachers, and state departments of education to discuss their implementation. Delegates to the conferences agreed that elementary school teachers were inadequately prepared to teach mathematics. The National Association of State Directors of Teacher Education and Certification, as well as all state directors of certification, have approved the CUPM recommendations. (47:41)

Fisher (1966) made a survey of 117 colleges and universities, chosen at random from a list of 822 institutions in the Guide to Undergraduate Programs in Mathematics.

A comparison was made of the total semester hours of mathematics required for the pre-service preparation of elementary school teachers in 1960 and in 1965. In 1960, more than one-half of the institutions surveyed required

no content courses in mathematics, although most of them did require methods courses in mathematics. By 1965, only one-sixth of the institutions did not require a mathematics course, which indicated a significant increase in the amount of mathematics required. However, this increase was confined to the study of the real number system, with few courses in algebra and geometry being required. (48)

In an attempt to determine the status of the mathematics training of elementary school teachers in our colleges and the extent of the implementation of the CUPM recommendations, the Committee on the Undergraduate Program in Mathematics canvassed 911 colleges in 1966. The 887 replies received revealed that some progress had been made in the area of Level I recommendations. The number of colleges requiring no mathematics for prospective elementary school teachers dropped from 22.7% in 1962 to 8.1% in 1966. Five or more semester hours of mathematics were required in 50.1% of our colleges in 1966, compared with 31.8% in 1962. Many colleges stated that they were planning to increase their requirements in the near future. (49)

Summary of Related Research

There is a pressing need for more research in the teaching and learning of mathematics. The discovery method,

while recognized as a valuable technique, has not been conclusively proven to be the best method for teaching mathematics to children.

Most elementary school teachers and prospective elementary school teachers do not possess the mathematical competence and understanding needed to teach the contemporary mathematics curriculum. Teacher attitudes toward the subject itself leave much to be desired.

More individualization is needed in pre-service and in-service training. Teachers of elementary school mathematics need a unique type of preparation, with courses that are relevant to the materials they will be expected to use in the classroom. Programs for beginning teachers should differ from those for older teachers in content, methods of approach, and media.

More cooperative relationships should be fostered between colleges and universities and schools, with professors serving as consultants.

Although programs for the mathematics preparation of elementary school teachers have improved in colleges and universities during the past few years, they are still far from reaching the standards set forth by the Committee on the Undergraduate Program in Mathematics.

CHAPTER III

METHODOLOGY AND PROCEDURES

The research design includes both a descriptive survey and a classroom inventory. (See Appendix.)

The Survey

The sample. The target population for the study is the approximately 27,000 Massachusetts elementary school teachers.

A random sample, obtained by computer through the cooperation of James F. Baker, Assistant Commissioner of Education, and Jesse O. Richardson, Director of Research and Field Services at the Research and Development Center at Woburn, Massachusetts, contains every one-hundredth name drawn from the list of teachers employed in schools included in the Massachusetts Public School Directory. The teachers are classified by age and sex. Both large and small school systems are evenly dispersed over the state.

The original sample contained 269 teachers. The sample used in the survey contains 200 teachers, for the following reasons:

When the Massachusetts Public School Directory was checked for addresses, it was discovered that several of

the males included in the sample were principals. Apparently, every teacher serving in any capacity at the elementary level was included in the list.

At the time the sample was drawn, the only list of elementary school teachers available was for the 1967-68 school year. Therefore, since some of the teachers were nearing the compulsory retirement age, they were also excluded.

Additional teachers were excluded from the sample to be surveyed, because they taught in schools which were to be used for the classroom inventory.

Instrumentation. The survey is a three-page instrument, designed to gather data on the educational backgrounds of selected Massachusetts elementary school teachers, specifically to determine whether their training coincides with the Level I recommendations of the Committee on the Undergraduate Program in Mathematics of the American Mathematical Association. Questions also pertain to the types of methods courses studied in mathematics and whether these courses included any aspect of the teaching of contemporary programs through the use of discovery methods. Other information is solicited to discover whether school systems are providing courses or workshops specifically related to teaching mathematical concepts to children

through discovery methods.

Data also pertain to the extent to which the following recommendations currently mentioned in the literature are being implemented:

The mathematics laboratory is a much-publicized resource for the teacher of mathematics. The laboratory is a special room, equipped with a variety of mathematics materials, audio-visual aids, and manipulative devices, even computers, where children can work on individual or group problems not included in the regular classroom work. Ideally, a person trained in mathematics would be in charge of the laboratory on a full-time basis to aid both pupils and teachers.

Team teaching techniques are encouraged. This type of teacher cooperation enables teachers to plan lessons together for more effective teaching. Special skills can be utilized, and teachers can work in those areas of mathematics for which they are best trained and in which they have the most interest.

The Committee on the Undergraduate Program in Mathematics has also recommended that each elementary school have a mathematics specialist, a person with a degree in mathematics, whom teachers can consult concerning problems connected with the teaching of elementary mathematics.

Mathematics resource centers are being provided in some schools. These centers contain a wide variety of mathematics materials, manipulative devices, professional books on mathematics, etc., which the teacher can take to use in the classroom or for his own professional improvement. In many schools, there is one resource center (sometimes the library) where materials for all subject matter fields are kept.

In summary, the questionnaire contains items designed to discover just how many of the recommendations being

made for the improvement of the teaching of mathematics are being followed within randomly selected Massachusetts elementary schools.

Classification of the data. Data pertaining to the extent to which the CUPM recommendations are being implemented in selected Massachusetts elementary schools are placed in three categories:

1. A one-year course in the study of the real number system (6 semester hours).
2. A one-half year course in introductory algebra (3 semester hours).
3. A one-half year course in informal geometry (3 semester hours).

Since, in some instances, partial implementation may be evidenced, the requirements are analyzed individually or in combinations, as follows: recommendation 1 only; recommendation 2 only; recommendation 3 only; recommendations 1, 2, and 3; recommendations 1 and 2; recommendations 1 and 3; recommendations 2 and 3; and those having no training which can be classified in any of the above categories.

The number of teachers who have been able to take advantage of courses and workshops offered since graduation are tabulated, with courses and workshops being designated as dealing either with conceptual mathematics or with discovery methods of teaching mathematics.

A study is made of the number of schools including released time periods in their schedules. Many administrators are realizing that the elementary school teacher needs some time during the school day to devote to lesson preparation, personal study, consultations with school specialists, etc. To pursue these activities, teachers are allowed preparation periods during which they are released from their duties in the classroom.

Released time is particularly important for the teacher of elementary mathematics. Studies have shown repeatedly that elementary school teachers are lacking in knowledge of concepts, as well as in favorable attitudes toward the subject itself. With the advent of modern mathematics, the problem has been intensified. Many teachers have neither the time nor the inclination to work outside of school hours on professional improvement or preparation of lessons.

Since teachers of mathematics rely to a great extent upon the textbook both for the materials to be taught and the sequence for teaching, it is important to make a study of the textbooks being used. Although no one textbook should be followed verbatim, it is doubtful that many teachers consult other sources in their lesson preparations.

A review of the publishers and publication dates of

textbooks listed in the survey returns gives an indication of the type of mathematics that is being taught in the classroom. Newer textbooks, published during the past seven or eight years, are based to a high degree on the work done by the School Mathematics Study Group and reflect this group's emphasis on teaching mathematics through discovery methods. As demonstrated in Chapter II, it is questionable whether many elementary school teachers have had adequate formal preparation to handle this type of curriculum. In many instances, books and materials are purchased by administrators who are not aware of the problems involved and do not make provisions for proper introduction of the teacher to the use of these materials.

In-service courses, conducted by qualified personnel, in the teaching of specific concepts and principles through the use of the discovery method are essential. Workshops and courses which include a study of the work done by various mathematics study groups or show one or two lessons taught by the discovery method are not the solution to the problem.

Data concerning the availability of courses dealing with heuristic methods of teaching mathematics give some indication of how important this aspect of mathematics instruction is considered to be by those in charge of seeing that the curriculum achieves the desired objectives.

In the absence of training, the inevitable outcome is that teachers resort to traditional expository methods of teaching mathematics. Many teachers may even be entertaining the delusion that they are employing discovery methods, because they are using a textbook which emphasizes discovery, when, in reality they are using expository methods. Of course, it must be conceded that an element of discovery may be present even in an expository lesson, but this cannot be considered discovery in the pure sense as it has been interpreted by those advocating the use of the method. Bearing this in mind, teacher preferences are categorized as being one of the three following methods of teaching mathematics: expository, discovery, or a combination of both.

Other questions concerning seating plans (formal or informal) and types of lesson presentations preferred (formal or informal) are designed to give further insight into the kind of mathematics teaching being carried on.

Overall mathematics training and experience are summarized with data given concerning the number of mathematics courses, number of workshop experiences in conceptual mathematics and discovery methods of teaching mathematics, and number and kinds of degrees earned.

The extent to which innovations are being followed within selected Massachusetts elementary schools can be

determined by an examination of the data on the four modern innovations mentioned previously: the mathematics laboratory, the mathematics resource center for teachers, the mathematics specialist, and team teaching.

Data on the above aspects of mathematics instruction are compiled into tables, as follows:

- | | |
|------------|---|
| Table 1. | The Implementation of CUPM Recommendations
by Teachers Included in the Survey |
| Table 2. | Mathematics Courses Attended since
Graduation from College |
| Table 3. | Schools Providing Released Time |
| Table 4-a. | Publishers and Publication Dates of
Mathematics Textbooks |
| Table 4-b. | Number of Textbooks Used in Teaching
Mathematics |
| Table 5. | Teachers Having Formal Training in the Use
of Contemporary Mathematics Materials |
| Table 6. | Teacher Preferences (Methods of Teaching) |
| Table 7. | Overall Educational Background and
Mathematics Training |
| Table 8. | Use of Innovations in Teaching Mathematics |

This data compilation includes frequencies of responses, frequencies by categories, and percentages.

The Classroom Inventory

The classroom inventory attempts to corroborate the results of the survey. The subjects are 27 elementary school teachers in schools in Central Massachusetts included in the original sample.

Observations of mathematics lessons being taught in classrooms and interviews with the teachers supply data for conclusions as to whether the recommendations of the various mathematics study groups and committees are being implemented in the reality of the elementary classroom or whether mathematics is still being taught in the traditional manner.

The format of the classroom inventory as far as educational background and training of teachers are concerned is basically the same as for the survey. In addition, a checklist used by the researcher pinpoints pupil reactions to the lesson being taught, characteristics of the classroom which are an indication of the kind of learning taking place, and an evaluation of the methods being used in the teaching of mathematics.

Data obtained from the classroom inventory are compiled into tables, as follows:

Table 9. The Implementation of CUPM Recommendations
by Teachers Included in the Classroom
Inventory

Table 10.	Mathematics Courses Attended since Graduation from College
Table 11.	Schools Providing Released Time
Table 12.	Publishers and Publication Dates of Mathematics Textbooks
Table 13.	Teachers Having Formal Training in the Use of Contemporary Mathematics Materials
Table 14-a.	Teacher Preferences (Methods of Teaching Mathematics)
Table 14-b.	Methods Used in Lessons Observed
Table 15.	Overall Educational Background and Mathematics Training
Table 16.	Use of Innovations in Teaching Mathematics
Table 17.	Classroom Atmosphere
Table 18.	Comparison of the Findings

This data compilation includes frequencies of responses, frequencies by categories, and percentages.

Additional questions not included in the survey solicit teacher opinions concerning the Level I recommendations of the Committee on the Undergraduate Program in Mathematics, the availability of workshops concerned with the use of heuristic methods in the teaching of mathematics, and the superiority of expository and drill methods over heuristic methods of teaching and learning mathematics.

Testing of Hypotheses

The six hypotheses stated previously are tested for differences of percentages through the use of the one-tailed critical ratio (z) test statistic.

The following symbols are used to designate the proportions:

p = the sample proportion.

π = the population proportion.

The formula used is:
$$z = \frac{p - \pi}{\sigma_p}$$

CHAPTER IV

ANALYSIS OF DATA

Introduction

This chapter analyzes the data obtained from the survey and the classroom inventory. (See Appendix.)

Further analysis is made of items included in the survey and the classroom inventory which are not pertinent to the testing of the hypotheses, but which are important in the teaching of mathematics, such as educational innovations and classroom atmosphere.

The Survey

Two-hundred questionnaires were mailed and 167 replies were received. One-hundred twenty-four (74%) of these were usable. Forty-three were from teachers who were ineligible to complete the questionnaire, because they were not teaching mathematics. Among these were included speech and reading specialists, English teachers, retired teachers, teachers on sick leave, two school librarians, two physical education teachers, and one school nurse. This limitation was mentioned previously in Chapter III. Eighteen questionnaires were returned un-

opened, because no forwarding addresses were available.

The following statistics are derived from information obtained from the questionnaires:

Age of teachers - the range is 22 to 67 years; the median age is 31; the modes are 24 and 27 (9 teachers of each age); and the average age is 35.3.

Size of schools - the range is 71 pupils to 1150 pupils; the median number of pupils enrolled is 400; the mode is 350; and the average number of pupils enrolled is 420.7.

Size of classes - the range is 15 to 38 pupils; the median number of pupils per class is 28; the mode is 30; and the average number of pupils per class is 28.5.

Testing of hypotheses. Hypothesis Number One was stated: That less than ten per cent of the elementary school teachers in Massachusetts have studied the twelve semester hours of mathematics required to meet the CUPM (Committee on the Undergraduate Program in Mathematics) Level I recommendations.

The CUPM Level I courses recommended for the training of elementary school teachers in mathematics, as described previously, consist of:

1. A one-year course in the study of the structure of the real number system (6 semester hours).
2. A one-half year course in introductory algebra (3 semester hours).

3. A one-half year course in informal geometry (3 semester hours).

Teachers in the survey are graduates of 45 different colleges and universities. The data indicate that 20 of these educational institutions offer courses which meet the requirements of Recommendation No. 1; 20 meet the requirements of Recommendation No. 2; 19 meet the requirements of Recommendation No. 3; 1 meets the requirements of Recommendations No. 1 and No. 3; 3 meet the requirements of Recommendations No. 2 and No. 3; 1 meets the requirements of Recommendations No. 1 and No. 2; and 15 meet all three of the CUPM Level I course requirements. Since many of these teachers are not recent graduates, it is possible that more of the requirements are now being met at some of these institutions.

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number One.

Experimental hypothesis $H_1: \pi < 10\%$

Alternative hypothesis $H_2: \pi \geq 10\%$

The following symbols are used to designate the proportions:

p = the sample proportion.

π = the population proportion.

The formula used is:
$$z = \frac{p - \pi}{\sigma p}$$

The alpha level for the decision of significance is set to be $P < .05$. (P is used to denote probability.)

The findings relative to Hypothesis Number One are summarized in Table 1. The sample proportion is found to be 18%, which is higher than the hypothetical population proportion of less than 10%.

TABLE 1
THE IMPLEMENTATION OF CUPM RECOMMENDATIONS
BY TEACHERS INCLUDED IN THE SURVEY

Recommendations	Studied	%	Not Studied	%
No. 1, No. 2, & No. 3	22	18	102	82
No. 1	60	48	64	52
No. 2	41	33	83	67
No. 3	32	26	92	74
No. 1 & No. 2	6	5	118	95
No. 1 & No. 3	2	2	122	98
No. 2 & No. 3	7	6	117	94
None	50	40	74	60

A z of 3.08 is obtained. The probability of selecting a sample with a sample proportion of 18% from a population ($n = 124$) with a proportion of less than 10% is .0010. Since this figure of .10% falls far below the

5% level of significance, the experimental hypothesis that less than 10% of the elementary school teachers in Massachusetts have studied the twelve semester hours of mathematics required to meet the CUPM (Committee on the Undergraduate Program in Mathematics) Level I recommendations must be rejected.

Hypothesis Number Two was stated: That at least three-fourths of these teachers have never had a mathematics course of any kind since graduating from college.

The findings from the survey show that 60% of these teachers have not had a workshop or a college course in mathematics since graduating from college. (See Table 2.)

TABLE 2

MATHEMATICS COURSES ATTENDED SINCE
GRADUATION FROM COLLEGE

Year Graduated	Yes	No
1930 - 1939	9	4
1940 - 1949	5	6
1950 - 1959	15	13
1960 - 1969	21	51
Total	50	74
Per Cent	40	60

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Two. See Hypothesis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P \leq .05$.

The hypotheses are set up, as follows:

Experimental hypothesis	$H_1: \pi \geq 75\%$
Alternative hypothesis	$H_2: \pi < 75\%$

A z of -3.85 is obtained, yielding a probability of less than .0010 of selecting a sample with a sample proportion of 60% from a population ($n = 124$) with a proportion of 75% or larger. Since .10% falls below the 5% level of significance, and the probability is less than .10%, the experimental hypothesis that at least three-fourths of the elementary school teachers in Massachusetts have never had a mathematics course of any kind since graduating from college must be rejected.

Hypothesis Number Three was stated: That less than ten per cent of these teachers are allowed released time to pursue personal study, prepare lessons, consult with school specialists, etc.

The proportion of elementary school teachers being

allowed released time periods, according to the survey, is 26%. (See Table 3.)

TABLE 3
SCHOOLS PROVIDING RELEASED TIME

Released Time	No.	%
Provided	32	26
Not provided	92	74

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Three. See Hypothesis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis	$H_1: \pi < 10\%$
Alternative hypothesis	$H_2: \pi \geq 10\%$

A z of 6.15 is obtained. The probability of selecting a sample with a sample proportion of 26% from a population ($n = 124$) with a proportion of less than 10%

is less than .0010. This figure of .10% falls far below the 5% level of significance, so the experimental hypothesis that less than ten per cent of these teachers are allowed released time to pursue personal study, prepare lessons, consult with school specialists, etc. must be rejected.

Hypothesis Number Four was stated: That at least three-fourths of these teachers rely almost entirely upon one textbook when offering children mathematics instruction.

The survey findings indicate that 94% of the elementary school teachers in Massachusetts use one textbook in teaching mathematics. (See Tables 4-a and 4-b.)

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Four. See Hypothesis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis	$H_1: \pi \geq 75\%$
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Alternative hypothesis	$H_2: \pi < 75\%$
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A z of 4.87 is obtained. The probability of selecting a sample with a sample proportion of 94% from a

population ($n = 124$) with a proportion of 75% or larger is less than .0010. However, although this figure falls far below the 5% level of significance, the sample proportion of 94% falls within the range of the hypothetical population proportion of 75% or greater, so the experimental hypothesis that at least three-fourths of the elementary school teachers in Massachusetts rely almost entirely upon one textbook when offering children mathematics instruction must be accepted.

TABLE 4-a

PUBLISHERS AND PUBLICATION DATES
OF MATHEMATICS TEXTBOOKS

Publisher	1961-64	1965-70	Total
Addison-Wesley Co.	5	17	22
American Book Co.	0	1	1
Harcourt, Brace & World	1	10	11
Holt, Rinehart & Winston	2	11	13
Houghton Mifflin Co.	0	9	9
Laidlaw Bros.	0	14	14
Science Research Assoc.	4	5	9
Sadler Co.	0	1	1
School Math. Study Group	1	0	1
Scott, Foresman & Co.	17	4	21
Silver Burdett Co.	6	5	11
Singer Co.	0	1	1
Zerex	1	0	1
No one text			7
Not given			2
Total	37	78	124

TABLE 4-b
NUMBER OF TEXTBOOKS USED IN
TEACHING MATHEMATICS

No. of Textbooks	No.	%
One textbook	117	94
More than one textbook	7	6

Hypothesis Number Five was stated: That of the elementary school teachers using contemporary mathematics materials and textbooks in Massachusetts, less than ten per cent of them have had formal training in the use of these materials.

According to the survey data, 77% of the teachers have had formal training to use the contemporary mathematics materials and textbooks.

A summary of the data in Table 5 indicates whether the training, if any, was obtained through college courses, workshops (one-day workshops by publishers not included), or both.

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Five. See Hypothe-

sis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis $H_1: \pi < 10\%$

Alternative hypothesis $H_2: \pi \geq 10\%$

A z of 25.76 is obtained. This indicates that the probability of selecting a sample with a sample proportion of 77% from a population ($n = 124$) with a proportion of less than 10% is less than .0010. This figure falls far below the 5% level of significance, so the experimental hypothesis that less than 10% of the elementary school teachers using contemporary materials and textbooks in Massachusetts have had formal training in the use of these materials must be rejected.

TABLE 5

TEACHERS HAVING FORMAL TRAINING IN THE USE
OF CONTEMPORARY MATHEMATICS MATERIALS

Type of Training	No.	%
College courses	46	37.1
Workshops	19	15.3
Both	30	24.2
Neither	29	23.4

Hypothesis Number Six was stated: That at least three-fourths of these teachers adhere to the expository (as contrasted with heuristic) methods of teaching children mathematics.

The data in Table 6 indicate that 11% of the elementary school teachers in Massachusetts say that they teach mathematics through expository methods.

TABLE 6
TEACHER PREFERENCES

Method Preferred	No.	%
Expository	14	11.3
Heuristic	14	11.3
Combination of both	96	77.4

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Six. See Hypothesis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis	$H_1: \pi \geq 75\%$
Alternative hypothesis	$H_2: \pi < 75\%$

A z of -16.41 is obtained, giving a probability of less than $.0010$ of selecting a sample with a sample proportion of 11% from a population ($n = 124$) with a proportion of 75% or larger. The experimental hypothesis that at least three-fourths of the elementary school teachers in Massachusetts adhere to expository methods of teaching children mathematics must be rejected, because this figure falls far below the 5% level of significance.

Analysis of data in Table 7. The data in Table 7 concern the educational backgrounds and mathematics training of the elementary school teachers in the survey. Three have degrees in mathematics; 27 more have degrees in areas other than elementary education; and 5 have no degree. Fifty (40%) have graduate degrees, which seems to indicate that the sample includes several teachers who have an interest in attaining higher educational goals. This interest may account for the fairly high proportion (40% as compared with the hypothetical population proportion of 25% or less) of teachers who have had courses in mathematics since graduating from college.

Fifteen of these teachers have had no college courses of any kind in mathematics.

TABLE 7
OVERALL EDUCATIONAL BACKGROUND
AND MATHEMATICS TRAINING

Degrees and Courses	No.	%
Degree in Elementary Education	89	71.8
Degree in Mathematics	3	2.4
Degree in other areas	27	21.8
No degree	5	4.0
Masters in Education	47	37.9
Masters in Mathematics	2	1.6
Masters in other areas	1	.8
College Methods Courses in Math.	89	71.8
Math. Courses other than Methods	89	71.8
No Courses in College Math.	15	12.1
No Courses except Math. Methods	20	16.1
Workshops in Conceptual Math.	28	22.6
Workshops in Discovery Methods of teaching Math.	34	27.4

Most of the 89 teachers who had mathematics methods courses in college stated that the courses were in no way concerned with heuristic methods of teaching mathematics. In many cases, unsolicited comments expressed the opinion that the methods courses had not been helpful to the teachers when they were faced with the problem of teaching mathematics in the classroom.

Of the 58 teachers who attended mathematics workshops,

34 stated that the workshops were concerned with teaching mathematics through heuristic methods. Only 10 had ever studied a course in mathematics which was specifically concerned with discovery methods of teaching mathematics and which involved actual children in classroom situations.

Analysis of data in Table 8. Data appearing in Table 8 do not seem to give evidence of wide acceptance of educational innovations.

TABLE 8

USE OF INNOVATIONS IN
TEACHING MATHEMATICS

Type of Innovation	Yes	%	No	%
Math. specialist	27	22	97	78
Team teaching	11	9	113	91
Math. laboratory	5	4	119	96
Math. resource center	36	29	88	71

Only 27 teachers have access to a mathematics specialist whom they can consult on problems in mathematics instruction.

Only 11 teachers are in schools where team teaching techniques are employed. Not many schools in the sample seem to be using this means of taking advantage of teacher potential in the area of mathematics.

Although the mathematics laboratory is a much-publicized facility whose merits can not be denied, only 5 respondents teach in schools where laboratories are available. A practical explanation of this situation may be that school systems lack the funds or the space to offer this means of mathematics enrichment to their pupils and teachers.

Mathematics resource centers are more prevalent than mathematics laboratories, with 36 teachers having access to this facility.

The Classroom Inventory

The following statistics are compiled from information obtained through observations of and interviews with 27 elementary school teachers in their classrooms:

Age of teachers - the range is 22 to 60 years; the median age is 43; the modes are 26 and 51 (4 teachers of each age); and the average age is 40.6.

Size of schools - The range is 65 pupils to 600 pupils; the median number of pupils enrolled is 600; the mode is 600; and the average number of pupils enrolled is 429.3.

Size of classes - the range is 19 to 32 pupils; the median number of pupils per class is 25; the mode is 25; and the average number of pupils per class is 24.8.

Testing of hypotheses. The hypotheses are the same as those tested in the survey.

Hypothesis Number One was stated: That less than ten per cent of the elementary school teachers in Massachusetts have studied the twelve semester hours of mathematics required to meet the CUPM (Committee on the Undergraduate Program in Mathematics) Level I recommendations.

The CUPM Level I courses recommended for the training of elementary school teachers in mathematics, as described

previously, consist of:

1. A one-year course in the study of the structure of the real number system (6 semester hours).
2. A one-half year course in introductory algebra (3 semester hours).
3. A one-half year course in informal geometry (3 semester hours).

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number One. (See Table 9 below for data.)

TABLE 9

THE IMPLEMENTATION OF CUPM RECOMMENDATIONS
BY TEACHERS INCLUDED IN THE
CLASSROOM INVENTORY

Recommendations	Studied	%	Not Studied	%
No. 1, No. 2, & No. 3	0	0	27	100
No. 1	3	11	24	89
No. 2	3	11	24	89
No. 3	0	0	27	100
No. 1 & No. 2	0	0	27	100
No. 1 & No. 3	0	0	27	100
No. 2 & No. 3	1	4	26	96
None	21	78	6	22

The proportion of teachers included in the classroom inventory who have studied the CUPM Level I recommendations is 0%.

The following symbols are used to designate the proportions:

p = the sample proportion.

π = the population proportion.

The formula used is:
$$z = \frac{p - \pi}{\sigma_p}$$

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis $H_1: \pi < 10\%$

Alternative hypothesis $H_2: \pi \geq 10\%$

A z of -1.75 is obtained. Although the probability of selecting a sample with a sample proportion of 0% from a population ($n = 27$) with a proportion of less than 10% is .0401 (below the 5% level of significance), 0% falls within the range of the hypothetical population proportion of less than 10%. Therefore, the experimental hypothesis that less than ten per cent of the elementary school teachers in Massachusetts have studied the twelve semester hours of mathematics required to meet the CUPM Level I recommendations must be accepted.

Hypothesis Number Two was stated: That at least three-fourths of these teachers have never had a mathematics course of any kind since graduating from college.

The findings from the classroom inventory show the sample proportion to be 52%. Both workshops and college courses are included in this analysis of the mathematics courses studied since graduation from college. (See Table 10.)

TABLE 10

MATHEMATICS COURSES ATTENDED SINCE
GRADUATION FROM COLLEGE

Year Graduated	Yes	No
1930 - 1939	4	2
1940 - 1949	6	3
1950 - 1959	0	1
1960 - 1969	3	8
Total	13	14
Per Cent	48	52

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Two. See Hypothesis Number One for the formula and the explanation of

the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis $H_1: \pi \geq 75\%$

Alternative hypothesis $H_2: \pi < 75\%$

A z of -2.77 is obtained. The probability of selecting a sample with a sample proportion of 52% from a population ($n = 27$) with a proportion of 75% or larger is .0028. This figure of .28% falls far below the 5% level of significance, so the experimental hypothesis that at least three-fourths of the elementary school teachers in Massachusetts have never had a mathematics course of any kind since graduating from college must be rejected.

Hypothesis Number Three was stated: That less than ten per cent of these teachers are allowed released time to pursue personal study, prepare lessons, consult with school specialists, etc. (See Table 11.)

TABLE 11

SCHOOLS PROVIDING RELEASED TIME

Released Time	No.	%
Provided	7	26
Not provided	20	74

Twenty-six per cent of the teachers interviewed in the classroom inventory are allowed released time periods.

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Three. See Hypothesis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis	$H_1: \pi < 10\%$
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Alternative hypothesis	$H_2: \pi \geq 10\%$
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A z of 2.81 is obtained. The probability of selecting a sample with a sample proportion of 26% from a population ($n = 27$) with a proportion of less than 10% is .0025. This figure of .25% falls far below the 5% level of significance, so the experimental hypothesis that less than 10% of the elementary school teachers in Massachusetts are allowed released time to pursue personal study, prepare lessons, consult with school specialists, etc. must be rejected.

Hypothesis Number Four was stated: That at least three-fourths of these teachers rely almost entirely upon one textbook when offering children mathematics instruction.

All of the teachers observed used one textbook in the teaching of mathematics. As can be seen from a study of Table 12, most of the textbooks being used were published within the past five years.

TABLE 12
PUBLISHERS AND PUBLICATION DATES
OF MATHEMATICS TEXTBOOKS

Publisher	Date	No.
Addison-Wesley Co.	1965-68	8
Harcourt, Brace & World	1962	1
Laidlaw Bros.	1965-68	10
Silver Burdett Co.	1963	3
Science Research Assoc.	1965	2
Scott, Foresman & Co.	1966	2
Winston	1959	1

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Four. See Hypothesis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis	$H_1: \pi \geq 75\%$
Alternative hypothesis	$H_2: \pi < 75\%$

A z of 3.01 is obtained. The probability of selecting a sample with a sample proportion of 100% from a population ($n = 27$) with a proportion of 75% or larger is .0013. Although this figure of .13% falls far below the 5% level of significance, the sample proportion of 100% falls within the range of the hypothetical population proportion of 75% or larger. Therefore, the experimental hypothesis that at least three-fourths of the elementary school teachers in Massachusetts rely almost entirely upon one textbook when offering children mathematics instruction must be accepted.

Hypothesis Number Five was stated: That of the elementary school teachers using contemporary mathematics materials and textbooks in Massachusetts, less than ten per cent of them have had formal training in the use of these materials.

Data concerning college courses and workshops in mathematics are summarized in Table 13.

Seventy per cent of the teachers interviewed have had formal training to use the contemporary mathematics

materials and textbooks.

TABLE 13
TEACHERS HAVING FORMAL TRAINING
IN THE USE OF CONTEMPORARY
MATHEMATICS MATERIALS

Type of Training	No.	%
College courses	8	29.6
Workshops	6	22.2
Both	5	18.5
Neither	8	29.6

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Five. See Hypothesis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis	$H_1: \pi < 10\%$
Alternative hypothesis	$H_2: \pi \geq 10\%$

A z of 10.53 is obtained. The probability of se-

lecting a sample with a sample proportion of 70% from a population ($n = 27$) with a proportion of less than 10% is less than .0010. This figure falls far below the 5% level of significance, so the experimental hypothesis that less than 10% of the elementary school teachers in Massachusetts have had formal training in the use of contemporary mathematics materials and textbooks must be rejected.

Hypothesis Number Six was stated: That at least three-fourths of these teachers adhere to the expository (as contrasted with heuristic) methods of teaching children mathematics.

The data in Tables 14-a and 14-b indicate that 21 out of the 27 teachers observed teaching mathematics did not use the method they said they preferred. Without exception, 100% of these teachers used the expository method.

TABLE 14-a

TEACHER PREFERENCES

Method Preferred	No.	%
Expository	6	22
Heuristic	1	4
Combination of both	20	74

TABLE 14-b

METHODS USED IN LESSONS OBSERVED

Method Used	No.	%
Expository	27	100
Heuristic	0	0
Combination of both	0	0

A one-tailed critical ratio (z) test involving proportion is used to test Hypothesis Number Six. See Hypothesis Number One for the formula and the explanation of the symbols used.

The alpha level for the decision of significance is set to be $P < .05$.

The hypotheses are set up, as follows:

Experimental hypothesis	$H_1: \pi \geq 75\%$
Alternative hypothesis	$H_2: \pi < 75\%$

A z of 3.01 is obtained. The probability of selecting a sample with a sample proportion of 100% from a population ($n = 27$) with a proportion of 75% or larger is .0013. Although this figure of .13% falls far below the 5% level of significance, the sample proportion of 100%

falls within the range of the hypothetical population proportion of 75% or larger. Therefore, the experimental hypothesis that at least three-fourths of the elementary school teachers in Massachusetts adhere to the expository (as contrasted with heuristic) methods of teaching children mathematics must be accepted.

Analysis of data in Table 15. Table 15 includes data concerning the educational backgrounds and mathematics training of the elementary school teachers who were observed and interviewed in the classroom inventory.

TABLE 15
OVERALL EDUCATIONAL BACKGROUND
AND MATHEMATICS TRAINING

Degrees and Courses	No.	%
Degree in Elementary Education	21	77.8
Degree in other areas	6	22.2
Masters in Education	8	29.6
College Methods Courses in Math.	21	77.8
Math. Courses other than Methods	16	59.3
No Courses in College Math.	6	22.2
No Courses except Math. Methods	8	29.6
Workshops in Conceptual Math.	8	29.6
Workshops in Discovery Methods of teaching Math.	0	0.0

Twenty-one of these teachers have degrees in elementary education; 6 have degrees in other areas; and 8 of the 27 have graduate degrees.

Six of these teachers (22%) have had no college courses in mathematics; 8 have had no mathematics courses in college other than methods courses.

All of the 8 teachers who had attended mathematics workshops stated that these workshops were concerned with conceptual mathematics. None had ever studied a course which was specifically concerned with heuristic methods of teaching mathematics and which involved actual children in classroom situations.

Educational innovations. None of the four educational innovations mentioned previously (mathematics specialist, team teaching, mathematics laboratory, and mathematics resource center) were being implemented in the schools visited. (See Table 16.)

TABLE 16

USE OF INNOVATIONS IN TEACHING MATHEMATICS

Type of Innovation	Yes	%	No	%
Math. specialist	0	0	27	100
Team teaching	0	0	27	100
Math. laboratory	0	0	27	100
Math. resource center	0	0	27	100

Classroom atmosphere. Table 17 presents a view of the classroom atmosphere found in the schools in the inventory. In all instances except one, the classrooms were pleasant and attractive, with bulletin board displays and a good supply of supplementary mathematics materials available for individual enrichment. In spite of the fact that formal seating in rows and rigid discipline prevailed, the children, in most instances, volunteered answers frequently and appeared to be interested in the mathematics lesson.

TABLE 17
CLASSROOM ATMOSPHERE

Condition	Yes	%	No	%
Formal seating	23	85	4	15
Rigid discipline	15	56	12	44
Physical conditions conducive to learning	26	96	1	4

Comparison of the findings. Table 18 compares the findings from the survey and the classroom inventory.

TABLE 18
COMPARISON OF THE FINDINGS

Item	Survey	Classroom Inventory
Studied CUPM recs. 1, 2, & 3	18%	0%
No Math. courses since college	60%	52%
Released time provided	26%	26%
Teachers using one textbook	94%	100%
Formal training to teach contemporary Math.	77%	70%
Teachers using expository methods	11%	100%

Similarities can be noted in the proportion of teachers being allowed released time periods (26% each), in the proportion of teachers who have studied no mathematics courses since graduating from college (60% as compared with 52%), in the proportion of teachers using one textbook (94% as compared with 100%), and in the proportion of teachers having formal training to teach contemporary mathematics (77% as compared with 70%).

Disparities occur in two categories: the teachers who have studied CUPM Level I course requirements (18% as compared with 0%) and the teachers who use expository methods in the teaching of mathematics (11% as compared with 100%).

semester hours of mathematics required to meet the CUPM (Committee on the Undergraduate Program in Mathematics) Level I recommendations.

The CUPM Level I courses recommended for the training of elementary school teachers in mathematics, as described previously, consist of:

1. A one-year course in the study of the structure of the real number system (6 semester hours).
2. A one-half year course in introductory algebra (3 semester hours).
3. A one-half year course in informal geometry (3 semester hours).

In comparing the results of the survey and the classroom inventory, the survey sample proportion of 18% (teachers who have studied all three of the recommended courses) is found not to be representative of the hypothetical population proportion of less than 10%, while the classroom inventory sample proportion of 0% is found to be within the range of the hypothetical population proportion. (See Tables 1 and 9.) Thus, while the survey results seem to indicate that implementation of the CUPM Level I recommendations is taking place among more than 10% of the Massachusetts elementary school teachers, the classroom inventory results do not substantiate this conclusion. Furthermore,

the classroom inventory findings come very close to being significant at the 5% level (.0401), while the survey findings give a probability of only .0010.

The difference in findings may be related to the difference in the sizes of the two samples. The survey sample of 124 teachers is almost five times as large as the classroom inventory sample of 27 teachers. Therefore, the survey sample would be expected to yield more valid results.

However, the survey sample proportion is only 18% which does not indicate a very high degree of implementation of CUPM Level I recommendations. Even allowing for a wide margin of error, we can safely assume that implementation is occurring among less than 50% of the population of Massachusetts elementary school teachers. If this proportion is representative of what is happening in other states, then nationwide progress has not been very great over a period of nine years.

It should be pointed out that, although all three of the recommendations are not being implemented to a high degree, the best progress is being made among teachers in the survey on Recommendation No. 1, which is concerned with the study of the structure of the real number system. Sixty

(48%) of the elementary school teachers in the survey have met this requirement. Forty-one teachers (33%) have studied Recommendation No. 2 and thirty-two teachers (26%) have studied Recommendation No. 3.

These findings would also seem to indicate that the recommended courses are not all required courses at the institutions offering them. Since 48% of the teachers have studied Recommendation No. 1, this course is very likely a required mathematics course for elementary school teachers. The subject matter itself places this course in the same category with one which is often called "Modern Mathematics Concepts" and is usually included as a part of a methods course, sometimes taught in the education department and sometimes in the mathematics department.

If this is indeed the case, then these institutions must regard Recommendation No. 1 as the most important of all, despite the fact that all three recommendations were endorsed by experts in all areas of mathematics education.

The relative merits of the CUPM Level I recommendations were discussed with the teachers in the classroom inventory. None could recognize the need at this level for sophisticated algebra and geometry backgrounds. All agreed that what is needed is one worthwhile course such as that required under Recommendation No. 1, together with a relevant course in the discovery methods of teaching these con-

cepts. There was evidence of a great deal of trepidation on the part of several of these teachers concerning the possibility of these requirements being enforced for in-service teachers. Some even expressed their determination to leave teaching, if this happened.

Most teachers expressed the greatest concern for the need of improved mathematics methods courses at both pre-service and in-service levels. While most of them had studied courses or attended workshops dealing with contemporary mathematics concepts, few felt secure in the teaching of these concepts to children. As indicated in Tables 14-a and 14-b, all 27 teachers were using expository methods to teach "discovery" mathematics, although 1 teacher had said that she used the discovery method all of the time and 20 teachers had said that they used a combination of both methods.

Hypothesis Number Two. Hypothesis Number Two was stated: That at least three-fourths of these teachers have never had a mathematics course of any kind since graduating from college.

It is encouraging to discover that this hypothesis must be rejected both in the survey sample and in the classroom inventory sample. The proportion of elementary school teachers who have studied courses and workshops in

contemporary mathematics is 40% for the survey sample and 48% for the classroom inventory. (See Tables 2 and 10.)

The findings indicate that elementary school teachers are concerned with improving their mathematics backgrounds for more subject matter knowledge and proficiency in teaching.

Hypothesis Number Three. Hypothesis Number Three was stated: That less than ten per cent of these teachers are allowed released time to pursue personal study, prepare lessons, consult with school specialists, etc.

The rejection of this hypothesis in both instances indicates that the idea of released time periods for elementary school teachers is catching on with administrators. It is interesting to note that the sample proportion of teachers being allowed released time is identical in the survey and the classroom inventory, namely 26%. (See Tables 3 and 11.) This proportion may not seem to be very large, but it must be remembered that the concept of released time for elementary school teachers has been adopted fairly recently. Released time is made possible in many school systems by utilizing the services of teacher interns or teacher aides.

Although released time periods are not scheduled for

any particular purpose, they are a boon to the teacher of mathematics who needs extra time to prepare lessons, talk over common problems with colleagues, or consult with a mathematics specialist when one is available.

Hypothesis Number Four. Hypothesis Number Four was stated: That at least three-fourths of these teachers rely almost entirely upon one textbook when offering children mathematics instruction.

The sample proportion of teachers relying upon one textbook is 94% for the survey and 100% for the classroom inventory. Although the probabilities of less than .10% (survey) and .13% (classroom inventory) indicate that the chances of selecting two such samples as these from a population with a proportion of 75% or larger are very small, the findings do substantiate the hypothesis that the population proportion is greater than 75%. These figures may indicate that it is highly improbable that the population proportion would be as large as 94-100% but would be closer to 75%.

Why do most teachers follow one textbook religiously when teaching mathematics? The first explanation that comes to mind is a practical one. Most school systems purchase one mathematics book of a series per child and a manual for the teacher. This being the case, unless teachers possess personal copies of other mathematics text-

books for reference and use in the classroom, they may lack the initiative and the motivation to seek these materials elsewhere. This is one reason why a mathematics resource center for teachers in the building is advantageous.

Let us examine the situation from another viewpoint. Although mathematics educators stress the desirability of encouraging the classroom teacher not to be completely dependent upon a single textbook, Rappaport criticizes certain aspects of the contemporary mathematics textbooks which may lead to the conclusion that it is better to use only one textbook. He denounces the diversity of definitions and the lack of precision in modern textbooks and manuals which have been prepared not by a single agency, but by individuals and organizations striving for rapid changes in the mathematics curriculum. (50:223) Consequently, the teacher, confused when terms are defined differently in different textbooks, becomes dependent upon a single textbook and afraid to consult any other. (50:227)

Hypothesis Number Five. Hypothesis Number Five was stated: That of the elementary school teachers using contemporary mathematics materials and textbooks in Massachusetts, less than ten per cent of them have had formal training in the use of these materials. Again, the rejection of this hypothesis in both instances with a sample

proportion of 77% having training in the survey and 70% in the classroom inventory (See Tables 5 and 13) points to good progress in the preparation of elementary school teachers in the area of contemporary conceptual mathematics. Colleges and universities are offering courses, school systems are providing workshops, and teachers are taking advantage of these opportunities to improve their educational backgrounds in contemporary mathematics.

However, few of these courses and workshops emphasize the use of heuristic methods of teaching contemporary mathematics concepts to children in the classroom.

Hypothesis Number Six. Hypothesis Number Six was stated: That at least three-fourths of these teachers adhere to the expository (as contrasted with heuristic) methods of teaching children mathematics. Here, we find a marked difference in findings, with this hypothesis being rejected in the survey and accepted in the classroom inventory, as was the case with Hypothesis Number One.

The sample proportion of teachers in the classroom inventory who use expository methods in the teaching of mathematics is 100% compared with 11% in the survey. (See Tables 6, 14-a and 14-b.) In the interviews, 21 of the 27 teachers in the classroom inventory said that they used heuristic methods or a combination of heuristic and expository

tory methods. When observed teaching mathematics, all used only expository methods. This leads the researcher to conjecture whether the Hawthorne effect is at work among the teachers in the survey. Are they responding in the manner they feel is expected of them, because they realize that this is the educationally acceptable method for teaching mathematics?

Other factors point toward this deduction: the small percentage of teachers who say they have had training in the use of heuristic methods to teach contemporary mathematics; the large amount of drill work that these teachers indicate they find necessary; the complaints of many of these teachers that heuristic methods are ineffective with slow learners (contrary to research findings cited in Chapter II); and the necessity for formal lesson presentations because of large classes and lack of classroom space.

In summary, the findings from the survey and the classroom inventory indicate that implementation of all three CUPM Level I recommendations is not taking place at a desirable rate, but good progress is being made on Recommendation No. 1 which is concerned with the study of the real number system.

Teachers are well-prepared in conceptual mathematics but lack sound pedagogical preparation for teaching the

contemporary programs through the use of heuristic methods.

Released time periods are being recognized as a valuable part of the elementary school schedule.

Elementary school teachers do rely heavily upon one textbook in the teaching of mathematics, and this may be a practice that will continue until mathematics educators and curriculum experts reach some consensus about the definitions of terms used in the contemporary mathematics textbooks and provide uniformity in this area.

The new mathematics is still being taught in the traditional manner, for the most part, because that is the only method teachers know how to use. Many favor the use of the heuristic methods but lack the expertise to employ them.

Recommendations

CUPM Level I recommendations. Elementary school teachers are not the only ones who are questioning the merits of the requirements set forth by the Teacher Training Panel of the Committee on the Undergraduate Program in Mathematics.

The criticisms of Gerald R. Rising, Professor of Mathematics Education at the State University of New York at Buffalo, were discussed in Chapter I (pp. 8-9). To re-

capitulate on this point, he does not feel that CUPM Level I course requirements provide the answer to teacher training problems, because CUPM courses are sometimes substituted for methods courses by professors who find it easier to lecture on mathematics than to help prospective teachers develop effective mathematical pedagogy. (8:299)

Arthur Morley, Principal Lecturer in Mathematics at Nottingham College of Education, England, doubts the wisdom of proposing such substantial mathematics programs for generalists and feels that the Cambridge Conference Teacher Training Report puts too much faith in what can be accomplished by improved content courses in isolation. (17:59) He recognizes that a second difficulty with the report is the assumption that treatment of a topic at a more sophisticated level will help a prospective teacher to use an appropriate level of treatment in the classroom. He recommends more involvement of college students in mathematical activity, such as problem solving, in seminars of not more than 20 students, followed by individual investigations by the students. (17:61)

Herbert F. Spitzer, former Professor of Mathematics Education at the University of Iowa feels that the problem is not the content of the college courses in mathematics, but the manner in which the content is presented

either by the college textbook, the college professor, or both. (16:138) The following excerpt from a letter received from Dr. Spitzer sums up his feelings in the matter:

Many of the best teachers of elementary school mathematics that I knew had very little training in mathematics. The CUPM recommendations are in my opinion unrealistic (too many hours) and what is worse, the mathematics recommended has little relationship to elementary school mathematics...I'd like to see some mathematics books for elementary school teachers-to-be that would ring the bell the way science, history, music or literature books do.

Hans-Georg Steiner, Director of the Didactic Seminar of the Institute of Mathematics at Karlsruhe, Germany, who is regarded as one of the world's leading mathematics educators reflects his realization of the need for a new pedagogy in mathematics in the following words:

It's an educational shame that students have been permitted to leave their mathematics classroom without ever having experienced the beauty of mathematical constructions and patterns, the challenging elements of game and play, the intellectual satisfaction of tackling and solving a problem, the exactness that comes only from clarity of language and from correct logical processes. (51:444)

He stresses two cardinal principles to be followed in the teaching of the new mathematics:

1. The student should be involved as early as possible in the process of building mathematics.
2. Basic concepts should be related to familiar

realities. (51:444)

All of these mathematics educators are in agreement with the survey and classroom inventory findings that more emphasis should be placed on pedagogical approaches that will help the classroom teacher in the task of involving pupils in meaningful mathematical activities.

Thus, it would seem that a reevaluation needs to be made of the relevance of the courses recommended by the CUPM Teacher Training Panel. With the exception of Recommendation No. 1, these requirements do not seem to be answering the needs of the elementary school teacher in the teaching of mathematics. The real need seems to be for training in heuristic methodology.

The place of drill in contemporary mathematics teaching. The large amount of drill found necessary by teachers in the survey seems to indicate that many teachers are using drill as a technique for learning rather than as a tool for reinforcement.

Placing high value on rapid calculations may lead to rote learning rather than learning that centers around understanding. (52:627)

Among the hoped-for skills and competencies to be acquired through the contemporary mathematics curriculum are deductive reasoning and logical thinking. Computational skill is important, also, but pupils should strive

for competency only after they come to understand and appreciate the process they are studying.

If teachers continue to present contemporary mathematics programs in the traditional manner, they are defeating the aims and objectives of the organizations and individuals who have worked so long and so hard to improve the mathematics curriculum.

Some new approaches to teacher training in mathematics.

David M. Clarkson, Associate Director of the Madison Project, Syracuse University, New York, thinks we would do well to emulate the English who seem to be doing a better job than we are in producing teachers who are able to use heuristic methods in the classroom. He offers his own ideas on the kind of background necessary:

1. A thorough knowledge and deep appreciation of the subject matter under discussion. Without this knowledge a teacher may not feel free to accept the kinds of divergent thinking which children will prefer [sic] in an open-ended or heuristic situation.
2. An ability and willingness to listen to children. The ability is necessary since communication with children is not always easy and the willingness is necessary because it underlies a respect for the intellectual integrity of children's thinking.
3. A thorough knowledge and considerable experience with small group dynamics. Where I have seen others and where I have failed in using heuristic methods, part of the failure has been due to an insensitivity to what is happening in the group of children who are at work with me. A study of small group dynamics may seem pedestrian in the context of such an exalted discussion as the use of heuristics in the elementary classroom, but I suppose it is a practical detail to which one must attend. (See letter in Appendix.)

Dr. Clarkson developed a mathematics methods course emphasizing the laboratory approach with an important role given to problem solving. Children from 4 neighboring school districts, including a residential school for delinquents (ages 5-12) participated in 5 to 10 laboratory sessions. Theoretical discussions and work with materials preceded the laboratory sessions with the children. Students were encouraged to pursue in depth topics in which they were interested, in order to develop good methods of attack. Working with children of varying ages and abilities enabled the students to discover their preferences before entering a teaching assignment.

Following Dr. Clarkson's suggestion, let us review the Nuffield Project, a very effective program for teacher training now in operation in England. In 1965, the Nuffield Foundation sponsored a mathematics teaching project which required local educational authorities to provide a teachers' center for the in-service training of teachers in the project, later for other schools. By 1966, 100 centers had been set up in mathematics, science, or both. Today, Britain has about 300 of these centers, where groups of 20-24 teachers meet, under the guidance of a leader teacher, one afternoon and one evening a week. They are assisted by lecturers from the College of Education or other centers. (53:407)

Projects such as the Madison Project (described in Chapter I) and the Nuffield Project are individualized laboratory approaches to teaching contemporary elementary school mathematics.

A mathematics course at San Diego State College (California) was designed to meet the CUPM Level I requirement concerned with the study of the structure of the real number system (Recommendation No. 1). The course was taught by a team of professors from the education and mathematics departments. This type of instruction enabled the education professors to become acquainted with students before meeting them in methods classes. The professors from the education department knew what topics in the elementary school curriculum should be stressed. The presentation of the course in this manner resulted in a substantial increase in the number of students electing the course, greater understanding between the two departments about the mathematics content for elementary school teachers, and a marked improvement in attitude toward mathematics on the part of student teachers. (54:256-57)

A teacher tends to teach a subject as he himself was taught. Teaching through lecture methods produces teachers who will tend to explain rather than provide classroom situations that will lead to understanding. Therefore, if we

want teachers to use heuristic methods of teaching, these teachers must be taught heuristically themselves. They must learn to set up concrete situations for themselves and manipulate them themselves. They need to learn techniques for organizing children into groups and getting them to work on mathematics problems. (55:265)

William R. Arnold teaches both content and method courses for elementary education majors at Colorado State College, Greeley, Colorado. He suggests that prospective elementary school teachers examine scope and sequence charts to determine which concepts can and cannot be taught through discovery methods. This helps students to discover strategies for teaching, because they will have some idea about what can be discovered by pupils in the classroom and what must be explained by the teacher. (56:570) The researcher recommends that other colleges and universities consider using the objectives of the mathematics methods course at Colorado State College as a model. They are summarized by Dr. Arnold, as follows:

1. Teach the prospective teacher as we would have him teach.
2. Teach the content of elementary mathematics in terms of having the pre-service teacher develop skill in the functional use of properties.
3. Show the prospective teacher how to prepare and teach lessons that call for effective combinations of explaining, discussing, and exploring.

4. Set an example conducive to building favorable attitudes in our students.
5. Make use of the psychology of number.
6. Use behavioral objectives and evaluate the learners in terms of those objectives.
7. Let the prospective teacher learn teaching strategies that will enable his students to learn via discovery.

Possibly, there are more effective ways of dealing with teacher training problems in mathematics than through high-level committee action. Theorizing and philosophizing about content requirements do not solve the problems that accompany the transference of the acquired knowledge into suitable classroom procedures.

The elementary school teachers are the ones who are facing the problems connected with the teaching of contemporary mathematics - why not give them a voice in the planning of programs that will furnish them with the help they need and want? If this planning took place at the local level, individual problems could be studied and remedied at the source.

Teacher centers such as those now operating so successfully in England might be established, with consultants from the mathematics and education departments of nearby colleges and universities acting as advisers. This kind of rapport might also result in more worthwhile methods

courses being taught at the institutions involved, since the professors would be cognizant of the kinds of problems that are connected with the teaching of contemporary mathematics in the elementary school classroom.

Summary. A reevaluation should be made of the CUPA Level I requirements to analyze the merits of this number of hours (12) of mathematics at a sophisticated level for generalists.

In addition, alternative solutions should be sought to the teacher training problem, with special attention being given to new pedagogical approaches.

Mathematics textbooks should be revised to obtain uniformity and precision in definitions of mathematical terms.

Emphasis should be placed on teaching elementary school mathematics through the use of heuristic methods. To accomplish this goal, teachers themselves should be taught heuristically.

Hardwick, Massachusetts
March 24, 1970

Dear Colleague:

The enclosed questionnaire represents an area of research in which you, as an elementary teacher of mathematics, have a vital interest. For this reason, I hope that you will be willing to cooperate with me in an effort to gather data pertaining to the mathematics backgrounds of elementary school teachers, as well as the kinds of textbooks and materials being used and the methods being employed to teach mathematics.

The researcher feels strongly that more and better training should be provided for the elementary school teacher in methods of teaching contemporary mathematics both in pre-service training at the undergraduate level and in in-service training at the graduate level. Therefore, it is necessary to compile significant statistical data to indicate that this improvement is essential, if the aims and objectives that have been set forth for the teaching of contemporary mathematics are to be achieved.

In completing the questionnaire, please keep in mind the intent of the following terms:

Contemporary Mathematics - A more precise term for "Modern" Mathematics, indicating the kind of mathematics that is being taught in the United States in the second half of the twentieth century.

Discovery Method - The following format is followed:

1. Hypotheses are made by teacher, pupil, or both.
2. Evidence, based on previous knowledge and experience, is presented to confirm or disconfirm the hypotheses.
3. The pupil is led to "discover" for himself the item of knowledge which is a warranted inference from steps 1 and 2.

Expository Method - The traditional method of teaching mathematics. The teacher explains or describes the principles, laws, or concepts to be learned and the pupil works out appropriate problems or examples to indicate his grasp of the knowledge being presented.

(2)

Conceptual Mathematics - A study of the basic concepts, laws, principles, patterns, sequences, ideas, and structure of mathematics. This type of study does not include methods of teaching the subject matter of mathematics.

All data supplied will be kept absolutely confidential and will be used only by the researcher in the compilation of mass statistics.

Thank you in advance for your time and cooperation. An early reply will be greatly appreciated, since time is an important factor in the completion of the survey.

Gratefully,

Mildred L. Vinskey
Graduate Student
University of Massachusetts

NAME OF TEACHER _____ AGE _____
SCHOOL SYSTEM _____ NAME OF SCHOOL _____
NO. OF PUPILS IN SCHOOL _____ NO. OF PUPILS IN YOUR CLASS _____
TOTAL NO. OF YEARS OF TEACHING EXPERIENCE _____
TOTAL NO. OF YEARS OF TEACHING EXPERIENCE IN THIS SCHOOL _____

EDUCATIONAL BACKGROUND

NAME OF COLLEGE OR UNIVERSITY ATTENDED _____
NO. OF YEARS ATTENDED _____ DEGREE & SUBJECT _____
YEAR GRADUATED _____

GRADUATE DEGREES

NAME OF COLLEGE OR UNIVERSITY ATTENDED _____
DEGREE & SUBJECT _____ YEAR GRADUATED _____

Check any of the following mathematics courses studied for the specified time at the college level:

1. A one-year course in the study of the real numbers _____
2. A one-half year course in introductory algebra _____
3. A one-half year course in informal geometry _____

List any other college courses in conceptual mathematics which you have studied. Give approximate name of course.

Have you had a college course in the methods of teaching mathematics? _____

If so, give the title of this course _____

Was any portion of this course devoted specifically to the use of discovery methods of teaching mathematics? _____

If so, please explain _____

Have you studied any courses specifically concerned with discovery methods of teaching mathematics which involved actual children in classroom situations? If so, please explain in detail: _____

Check, if you have ever attended a workshop or institute concerned with either or both of the following two areas:

Conceptual mathematics _____

Discovery methods of teaching mathematics _____

Is there a mathematics specialist available with whom you can discuss problems concerning the teaching of mathematics? _____

Are you allowed released time during the school day to pursue personal study, prepare lessons, or consult with school specialists? _____

Does your school utilize team teaching techniques? That is, does the school have a planned program providing for teachers to meet periodically to discuss common problems in the teaching of mathematics and to plan lessons together, with a frequent interchange of classes to allow teachers to instruct in those areas in which they are most competent or most interested? If so, please explain: _____

Check, if either of the following are available in your school:

Mathematics Laboratory (a room supervised by a qualified mathematics teacher and equipped with mathematics materials and manipulative devices, where pupils can work on individual or group problems not included in the regular classroom work.) _____

Resource Center, where teachers can obtain audio-visual aids for mathematics teaching, supplementary mathematics enrichment materials, mathematics reference books, etc.

List any objects or manipulative devices which you use from time to time in the teaching of mathematics: _____

What is the title of the mathematics textbook you use? _____

_____ Publisher _____

Copyright Date _____ Do you use a textbook? _____ If so, give

Publisher _____ Copyright Date _____

List any other supplementary materials you use: _____

Describe the seating arrangement of your room: _____

Do you favor formal or informal presentations in the teaching of mathematics? Formal _____ Informal _____ Please give reasons for your preference: _____

Approximately what percentage of your teaching do you find necessary to devote to drill work? _____

List areas in which you find it necessary to use persistent drill (either oral or written): _____

Do you favor using discovery methods or expository (tell and do) methods, or a combination of both, in the teaching of mathematics? Please explain: _____

List advantages, if any, that you have been able to observe in the manner in which presentations are made in the newer textbooks: _____

List disadvantages: _____

Has the school system in which you teach made provisions for the in-service training of teachers in contemporary mathematics? _____

If so, how many such workshops have you attended?

Who conducts these workshops?

How have you benefited personally from attendance at these workshops? (Use other side of paper, if necessary.)

CLASSROOM INVENTORY

NAME OF TEACHER _____ AGE _____
 SCHOOL SYSTEM _____ NAME OF SCHOOL _____
 NO. OF PUPILS IN SCHOOL _____ NO. OF PUPILS IN CLASS _____
 TOTAL NO. OF YEARS OF TEACHING EXPERIENCE _____
 TOTAL NO. OF YEARS OF TEACHING EXPERIENCE IN THIS SCHOOL _____

EDUCATIONAL BACKGROUND

NAME OF COLLEGE OR UNIVERSITY ATTENDED _____
 NO. OF YEARS ATTENDED _____ DEGREE & SUBJECT _____
 YEAR GRADUATED _____

GRADUATE DEGREES

NAME OF COLLEGE OR UNIVERSITY ATTENDED _____
 DEGREE & SUBJECT _____ YEAR _____

LIST ALL MATHEMATICS COURSES STUDIED:

<u>APPROXIMATE NAME OF COURSE</u>	<u>LENGTH OF COURSE</u>
_____	_____
_____	_____
_____	_____
_____	_____

Have you had a college course of any kind in methods of teaching mathematics? _____

If so, what was the approximate title of this course? _____

(2)

Did this course deal in any way with heuristic (discovery) methods of teaching mathematics? _____

Have you studied any courses specifically concerned with discovery methods of teaching and learning mathematics? _____

Did this courses (or courses) deal with actual children in classroom situations? Please give details: _____

Have you ever attended a workshop or institute concerned with either or both of these two areas?

Conceptual mathematics _____

Heuristic methods of teaching mathematics _____

Is there a mathematics specialist available in your school with whom you can discuss problems concerning the teaching of mathematics? _____

Are you allowed released time during the school day to pursue personal study, prepare lessons, or consult with school specialists? _____

Does your school utilize team teaching techniques? _____

(3)

ITEMS TO BE CHECKED BY RESEARCHER DURING CLASSROOM
OBSERVATION

A relaxed atmosphere prevails in the classroom _____

Rigid discipline is stressed _____

Seating is formal _____

Seating is informal _____

Children work in groups _____

Children are engaged in individual endeavors _____

The class is taught as a unit _____

There is a combination of class work and group work _____

Children are enthusiastic and volunteer answers
frequently _____

Children volunteer answers frequently but do not give
evidence of enthusiasm _____

Children are attentive but do not participate in the dis-
cussion _____

Children are inattentive, but quiet _____

Children are inattentive and noisy _____

The classroom atmosphere is conducive to learning _____

The classroom atmosphere is not conducive to learning _____

There is a number line exhibited in the classroom _____

Geoboards are available _____

The school has a mathematics laboratory _____

The school has a teacher resource center _____

The following manipulative devices and materials are available:

An abacus _____

Cuisenaire rods _____

Mathematics games _____

Computers (manual) _____

Geometric shapes _____

Books containing mathematics related stories, puzzles, etc. _____

Others: _____

The following visual aids are available either in the classroom or elsewhere in the school:

Movie projector _____

Film-strip projector _____

Overhead projector _____

Opaque projector _____

Overhead or portable screen _____

The teacher is using a textbook published within the last five years _____ Publisher _____ Copyright Date _____

The teacher is using a workbook published within the last five years _____ Publisher _____ Copyright Date _____

Other contemporary materials being used: _____

The teacher writes her own materials _____

The teacher uses materials prepared by a teacher team in the school _____

The children prepare some of their own materials _____

The lesson observed was taught through expository methods _____

The lesson observed was taught through heuristic methods according to the following format:

1. Hypotheses were made by teacher, pupil, or both.
2. Evidence was presented to confirm or disconfirm the hypotheses.
3. Pupils stated the item of knowledge which was a warranted inference from steps 1 and 2.

The lesson observed was taught through a combination of expository and heuristic methods _____

Comments expressed in interview with teacher observed on following items:

1. Amount of education in conceptual mathematics required by CUPM recommendations:
2. Available training in heuristic methodology:
3. Attitude toward superiority of expository and drill methods over heuristic methods of teaching and learning mathematics:

Copy of letter received from Dr. Herbert F. Spitzer
1602 Ranch Road
Georgetown, Texas 78626

Dear Mrs. Vinskey:

Sorry that this request was so long in reaching me. I left the University of Iowa in June 1968 and have since taught only two classes at the University of Texas. So, you see I'm semi-retired.

I'm interested in your problem but will not be of any help. I left Iowa because we could not get any help from the Mathematics department or anyone else on that very problem.

Many of the best teachers of elementary school mathematics that I knew had very little training in mathematics. The CUPM recommendations are in my opinion unrealistic (too many hours) and what is worse, the mathematics recommended has little relationship to elementary school mathematics. I thought the University of Texas situation (6 hours of mathematics and 2 hours of methods and the latter elective) might point toward a solution. I was very disappointed in what I found, and they have a good sincere teacher of the required mathematics. The students worked hard on the mathematics - just to pass the tests with a high mark - and then tried to forget it all. As a result, the students in my methods classes actually knew very little of the mathematics needed to understand arithmetic.

So, you see why I can't offer any minimum standard. I'd like to see some mathematics books for elementary school teachers-to-be that would ring the bell the way science, history, music or literature books do.

Sincerely,

(Signed) Herbert F. Spitzer

Copy of letter received from Dr. David M. Clarkson
Associate Director
The Madison Project
Syracuse University
Syracuse, New York

November 19, 1969

Mrs. Mildred Vinskey
Sessions Road
Hardwick, Mass. 01037

Dear Mrs. Vinskey:

I have discussed our recent telephone conversation with Dr. Davis who is very interested in the subject of your doctoral thesis and will send you a statement of his viewpoints on the training of elementary teachers to use heuristic methods. I would think that something might be gained from examination of a collection of such statements from many mathematics educators and indeed may I suggest that you solicit one from David Wheeler, who is the editor of Mathematics Teaching (official journal of the Association of Teachers of Mathematics in England) and is working with Caleb Gattegno at Schools for the Future in New York. Another person who could certainly make an important statement to you on this topic would be Her Majesty's Inspector of Schools, Miss Edith Biggs (H.M.I.) who lives at 2 Carlton Gardens, Ealing, London, W.5, England.

In fact, the English seem to be doing a better job than we are doing in producing teachers who are able to use heuristic methods in the classroom. A third English educator who could give you a significant statement of background requirements would be Arthur Morley, Department of Mathematics, Nottingham College of Education, Nottingham, England.

My own feeling as I told you on the phone is that the problem of preparing teachers to use heuristic methods requires considering the prior problem of how good teachers already use heuristic methods in the classroom. It is clear that some use them more effectively than others and, in my observations of good heuristic teaching in elementary classrooms, I have found considerable diversity of approach.

For example, I have seen good heuristics in fairly rigidly controlled classroom situations and I have seen poor heuristics in relatively free classroom situations, so that it seems that rigidity or looseness of classroom atmosphere is possibly irrelevant to the question of effective use of heuristic methodology. However, for what it's worth, here are my immediate ideas on what sort of a background an elementary teacher needs in order to use heuristic methods in the classroom:

1. A thorough knowledge and deep appreciation of the subject matter under discussion. Without this knowledge a teacher may not feel free to accept the kinds of divergent thinking which children will prefer in an open-ended or heuristic situation.

2. An ability and willingness to listen to children. The ability is necessary since communication with children is not always easy and the willingness is necessary because it underlies a respect for the intellectual integrity of children's thinking.

3. A thorough knowledge and considerable experience with small group dynamics. Where I have seen others and where I have failed in using heuristic methods, part of the failure has been due to an insensitivity to what is happening in the group of children who are at work with me. A study of small group dynamics may seem pedestrian in the context of such an exalted discussion as the use of heuristics in the elementary classroom, but I suppose it is a practical detail to which one must attend.

There are probably a lot of other things that a teacher using heuristic methods in the classroom needs to know, but I feel these three points are the major ones. Actually I wish I knew a lot more about it.

I hope this is of some help to you.

Sincerely yours,

(Signed) David M. Clarkson
Associate Director

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